Liquidity Constraints, Precautionary Saving, and Counterclockwise Concavification

Christopher D. Carroll

Miles S. Kimball

ccarroll@jhu.edu

Presentation in Copenhagen, June 2005

Substitutes or Complements?

Substitutes or Complements?

- Substitutes

Substitutes or Complements?

- Substitutes
  - If forced to save by LC's, PS may be less

Substitutes or Complements?

- Substitutes
  - If forced to save by LC's, PS may be less
  - If forced to save by PS, LC's may be irrelevant

Substitutes or Complements?

- Substitutes
  - If forced to save by LC's, PS may be less
  - If forced to save by PS, LC's may be irrelevant
- Complements

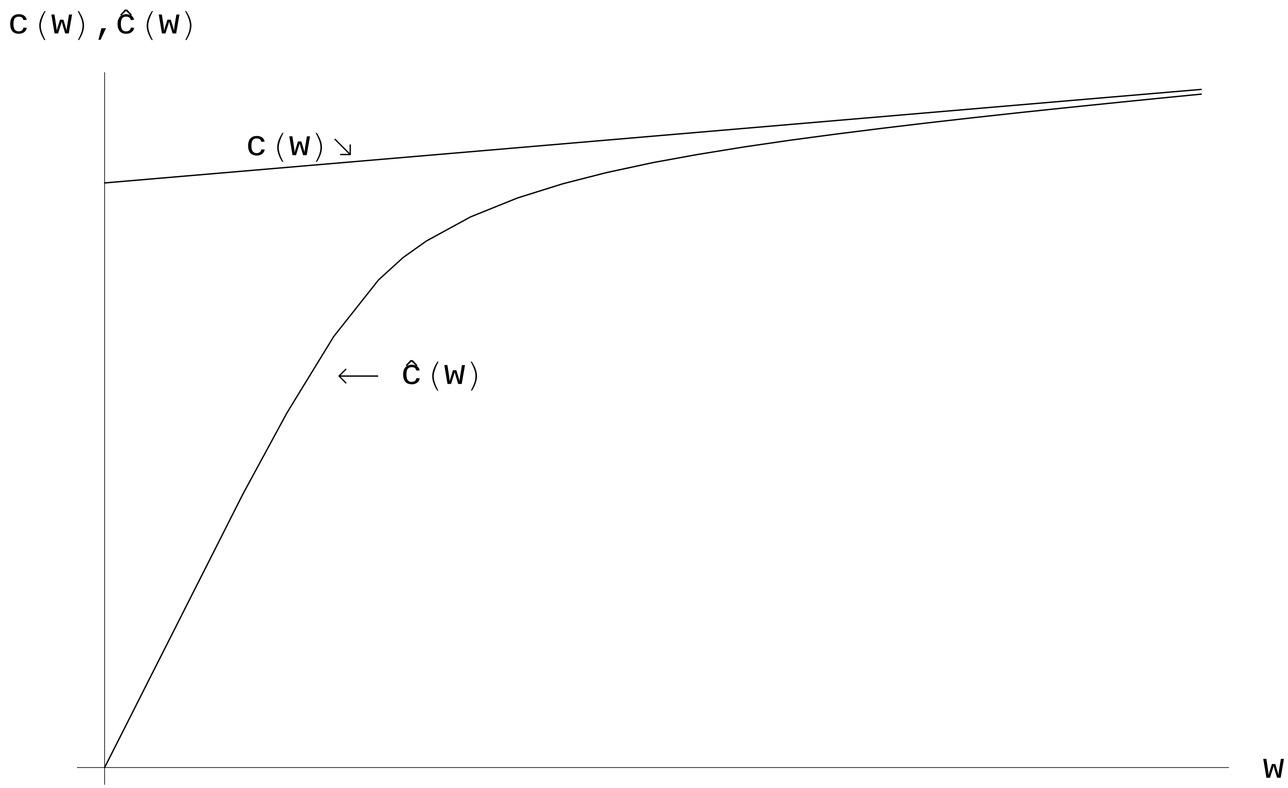
Substitutes or Complements?

- Substitutes
  - If forced to save by LC's, PS may be less
  - If forced to save by PS, LC's may be irrelevant
- Complements
  - Knowing you might be constrained intensifies PS

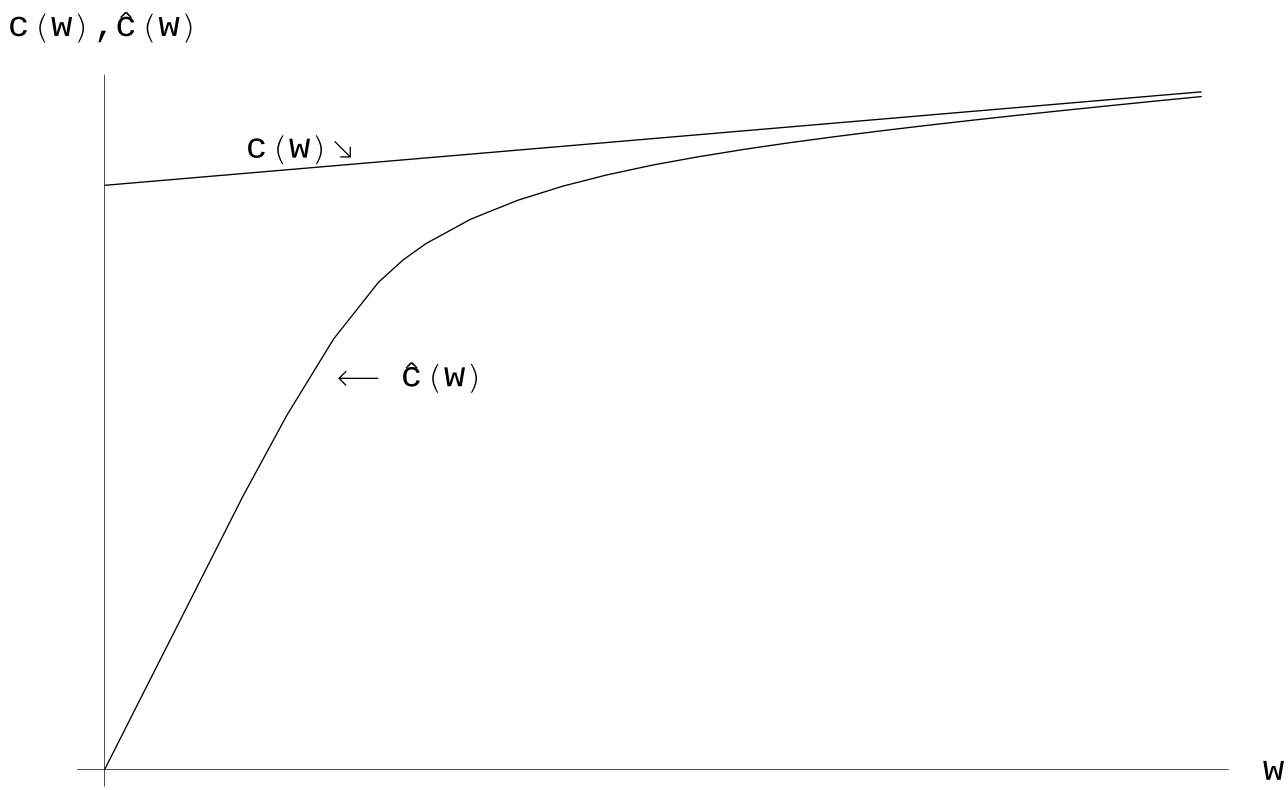
Substitutes or Complements?

- Substitutes
  - If forced to save by LC's, PS may be less
  - If forced to save by PS, LC's may be irrelevant
- Complements
  - Knowing you might be constrained intensifies PS
  - Knowing you might face risk intensifies LC effect .

Answer: It's All About Concavity

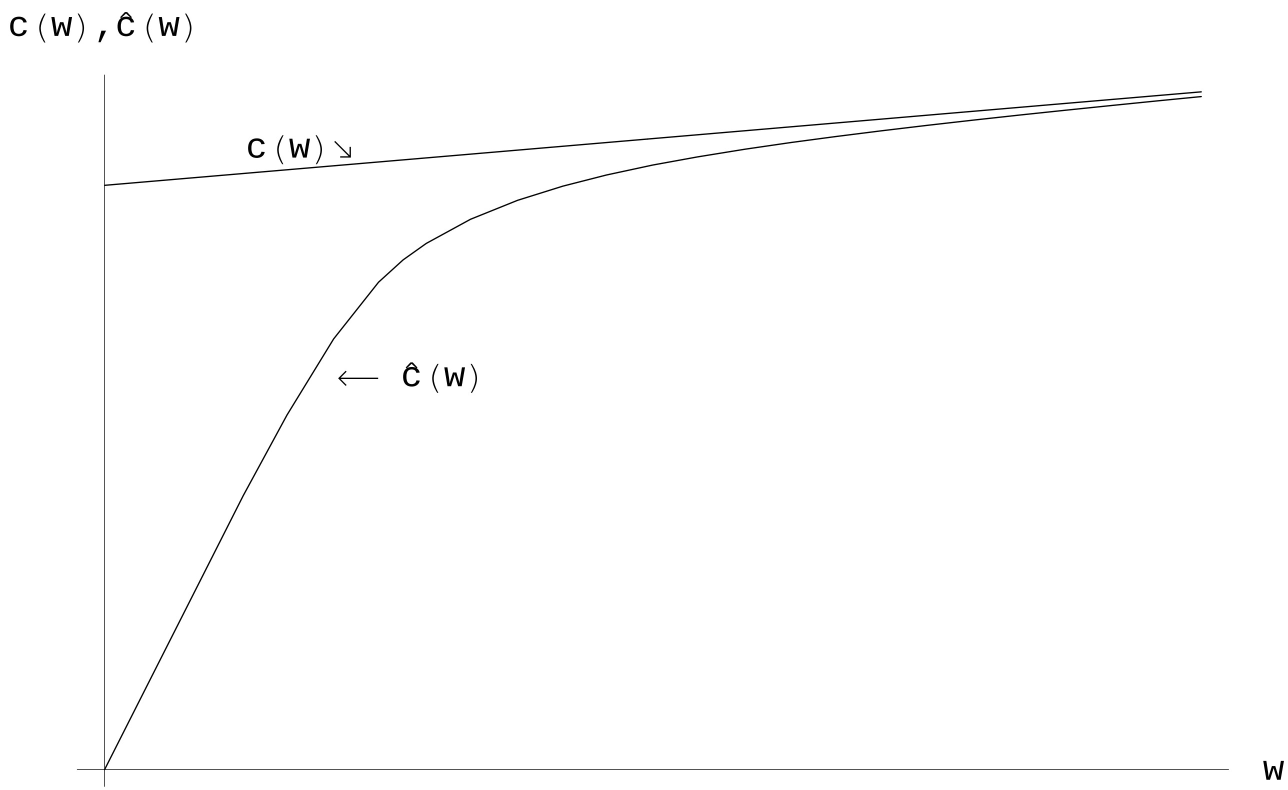


Answer: It’s All About Concavity



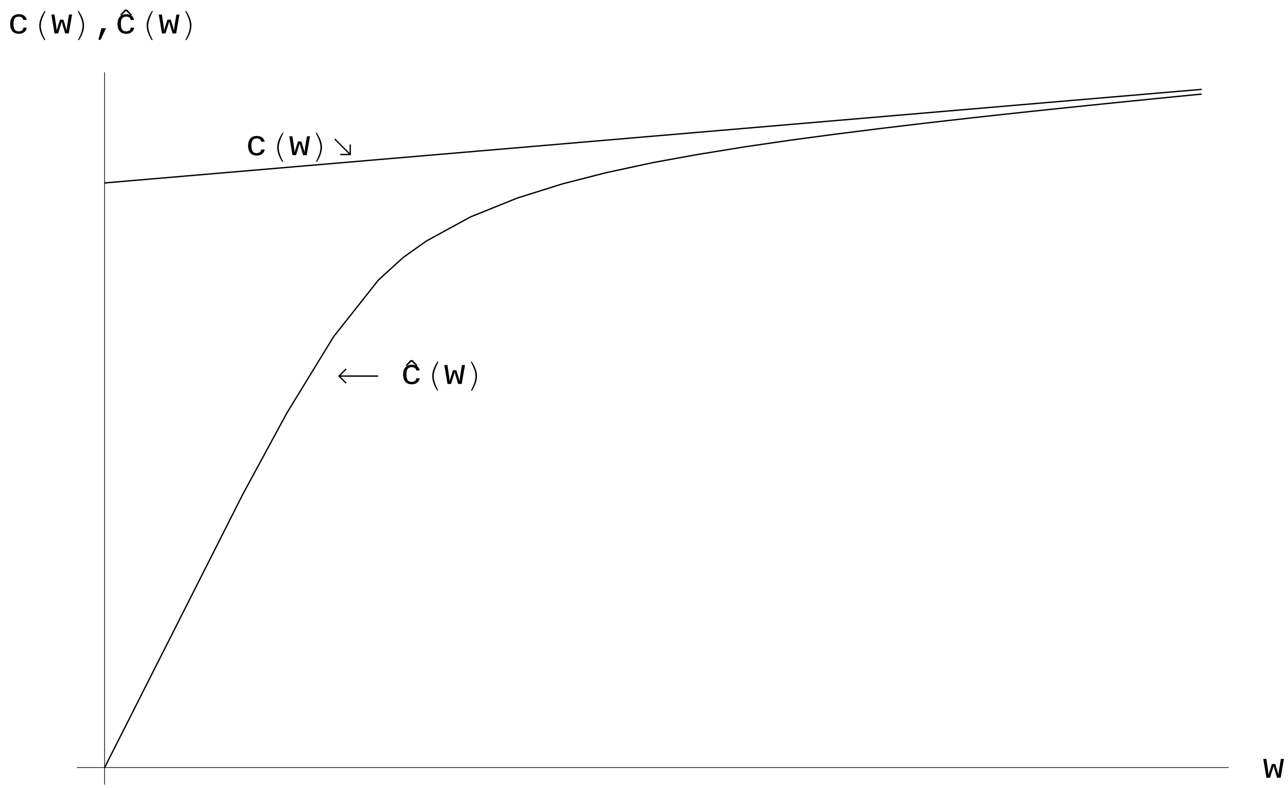
- PF Unconstrained Linear Baseline and ‘Modified’ Cases

Answer: It's All About Concavity



- PF Unconstrained Linear Baseline and 'Modified' Cases
- Poor=Young: Banks-Smith, Lusardi, Jappelli

Answer: It's All About Concavity



- PF Unconstrained Linear Baseline and 'Modified' Cases
- Poor=Young: Banks-Smith, Lusardi, Jappelli
- Rich: Flavin

Answer: It’s All About Concavity

PS, LC, and Concavity

PS, LC, and Concavity

- Constraints Induce Concavity

PS, LC, and Concavity

- Constraints Induce Concavity
  - With Low Wealth, MPC High Because Can't Borrow

PS, LC, and Concavity

- Constraints Induce Concavity
  - With Low Wealth, MPC High Because Can't Borrow
- PS Induces Concavity

PS, LC, and Concavity

- Constraints Induce Concavity
  - With Low Wealth, MPC High Because Can't Borrow
- PS Induces Concavity
  - With Low Wealth, MPC High Because of Fear

PS, LC, and Concavity

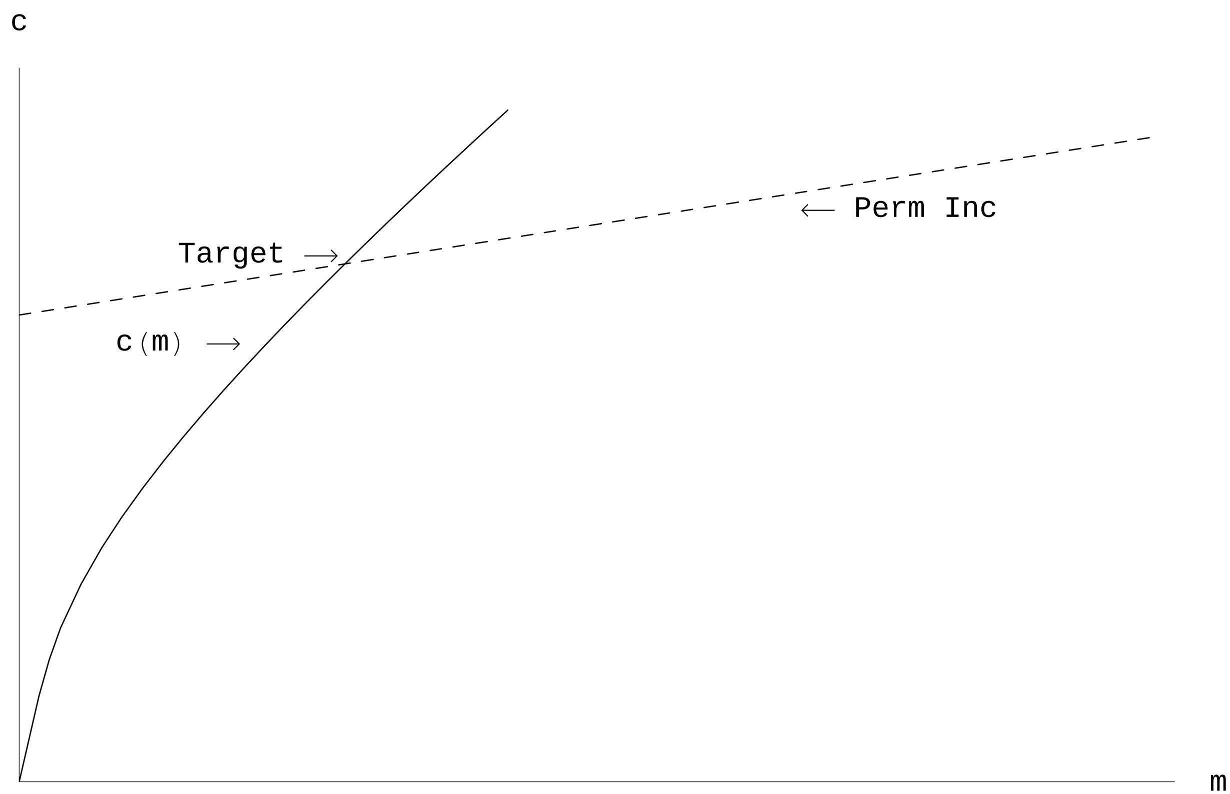
- Constraints Induce Concavity
  - With Low Wealth, MPC High Because Can’t Borrow
- PS Induces Concavity
  - With Low Wealth, MPC High Because of Fear
- PS/LC Interaction Depends on Effect on Concavity

PS, LC, and Concavity

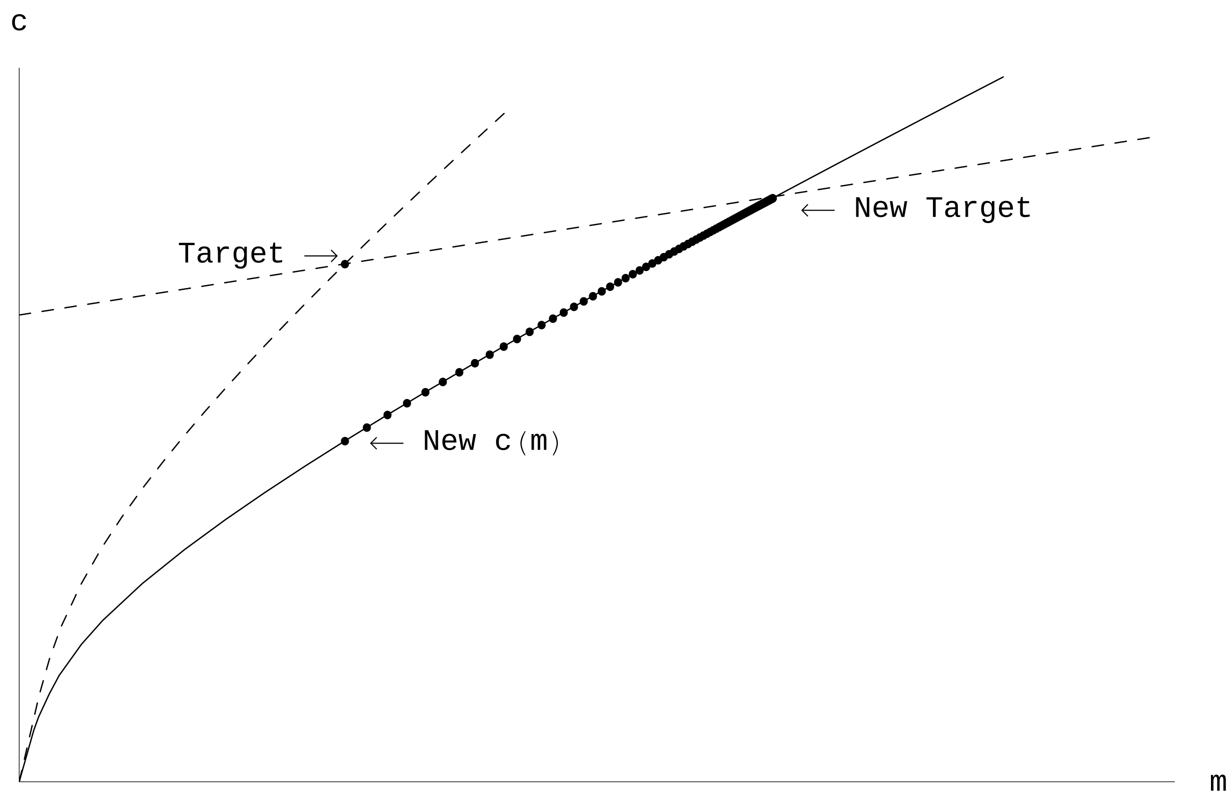
- Constraints Induce Concavity
  - With Low Wealth, MPC High Because Can't Borrow
- PS Induces Concavity
  - With Low Wealth, MPC High Because of Fear
- PS/LC Interaction Depends on Effect on Concavity
  - Concavity might go up at some m, down at others

Digression on Herrings

Digression on Herrings



Digression on Herrings



Digression on Herrings

- Lusardi: “What Is Your Target Wealth?”

Digression on Herrings

- Lusardi: “What Is Your Target Wealth?”
  -

Digression on Herrings

- Lusardi: “What Is Your Target Wealth?”
  - Hurd: “A bequest motive could extend the time horizon, reducing or eliminating any effects of mortality risk.”
  - Browning: What Do Focus Groups Say?

Digression on Herrings

- Lusardi: “What Is Your Target Wealth?”
  - Hurd: “A bequest motive could extend the time horizon, reducing or eliminating any effects of mortality risk.”
  - Browning: What Do Focus Groups Say?
  - Kennickell: People Have No Trouble Answering

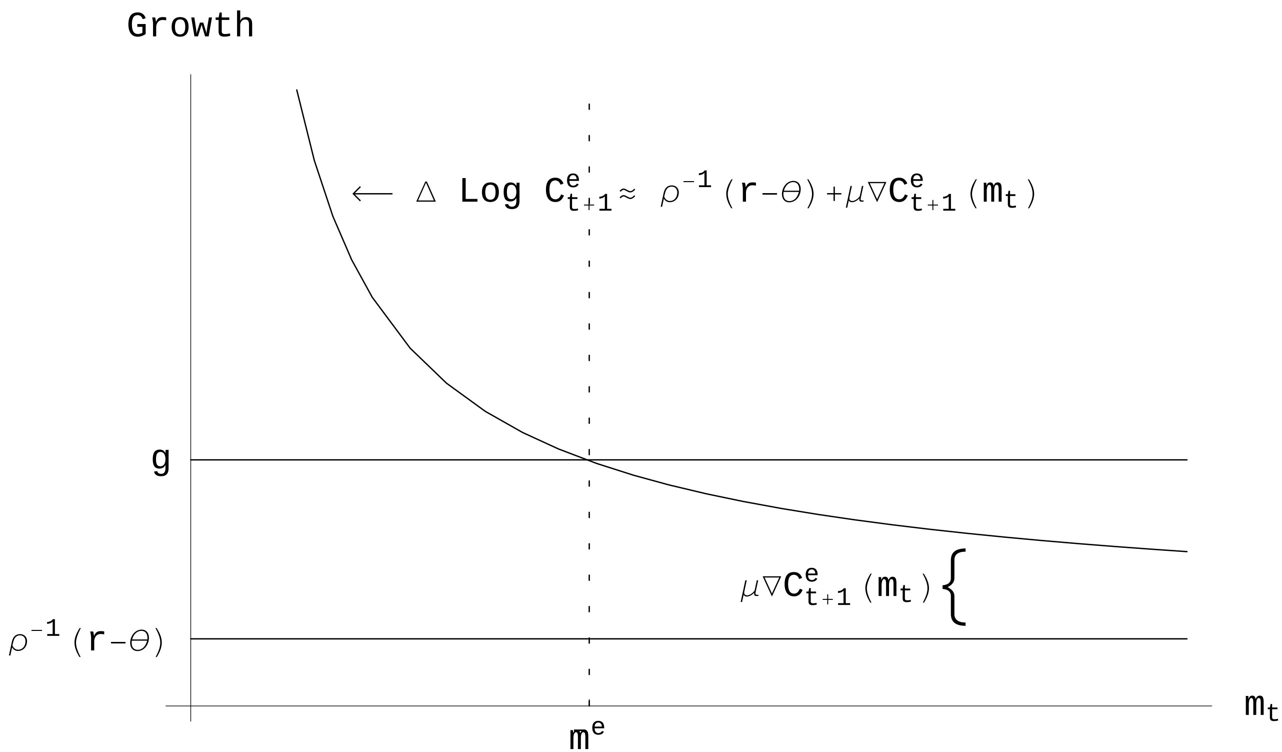
Digression on Herrings

- Lusardi: “What Is Your Target Wealth?”
  - Hurd: “A bequest motive could extend the time horizon, reducing or eliminating any effects of mortality risk.”
  - Browning: What Do Focus Groups Say?
  - Kennickell: People Have No Trouble Answering
- Some Herrings Are Red

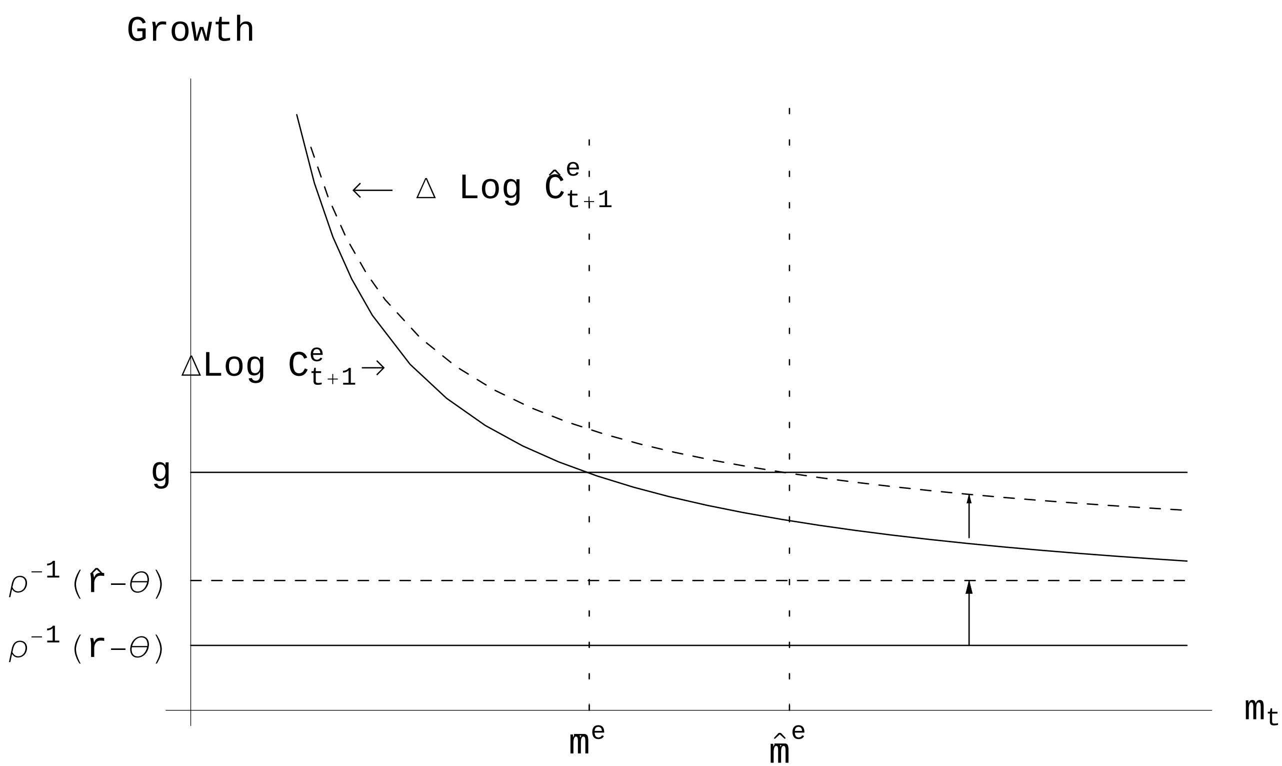
Digression on Herrings

- Lusardi: “What Is Your Target Wealth?”
  - Hurd: “A bequest motive could extend the time horizon, reducing or eliminating any effects of mortality risk.”
  - Browning: What Do Focus Groups Say?
  - Kennickell: People Have No Trouble Answering
- Some Herrings Are Red
- Some Are Not!

Digression: Target Wealth Ratio



Digression: Target Wealth Ratio



Prudence

Prudence

- Intensity of precautionary saving motive at a point

Prudence

- Intensity of precautionary saving motive at a point
- Kimball (1990) defines as

$$\mathbf{P}(c) = \left(\frac{-u'''(c)}{u''(c)} \right)$$

Prudence

- Intensity of precautionary saving motive at a point
- Kimball (1990) defines as

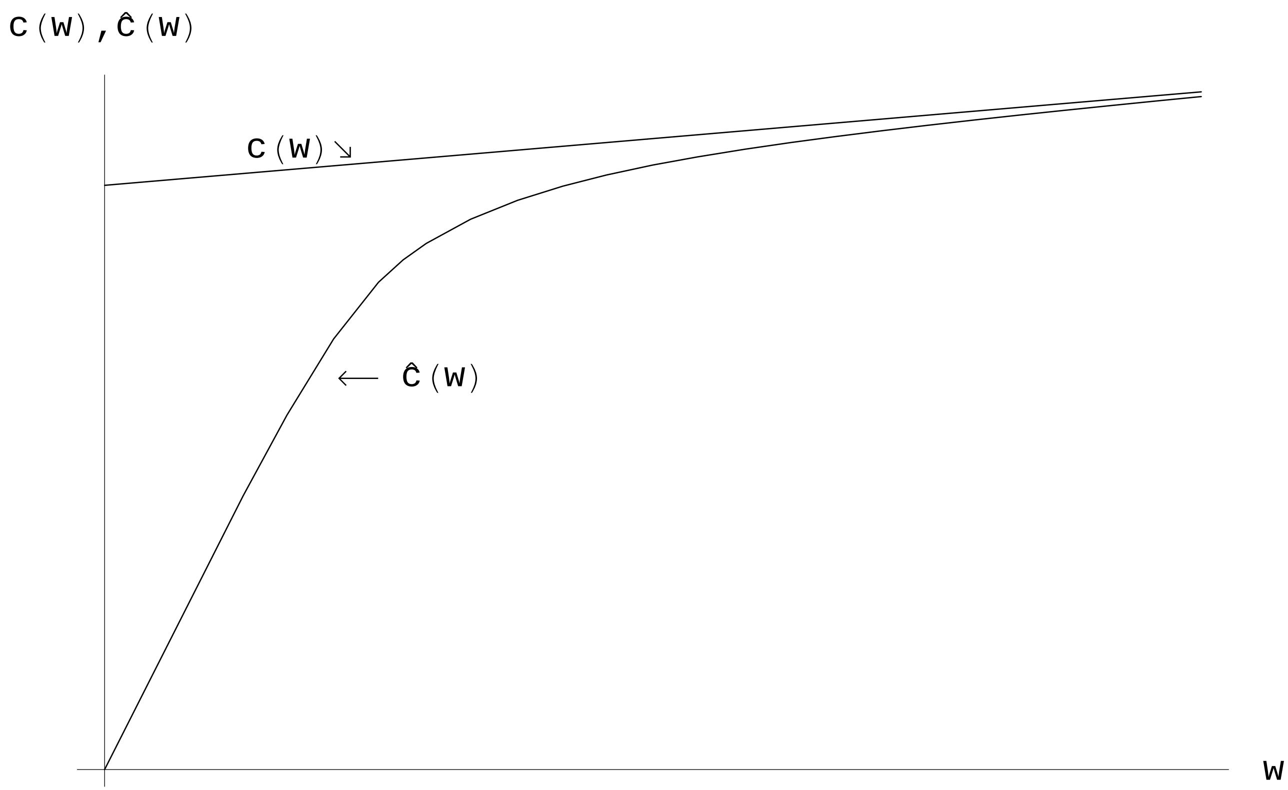
$$\mathbf{P}(c) \;=\; \left(\frac{-u'''(c)}{u''(c)}\right)$$

- Prudence of $\hat{V}(m)$ exceeds that of $V(m)$ at m if

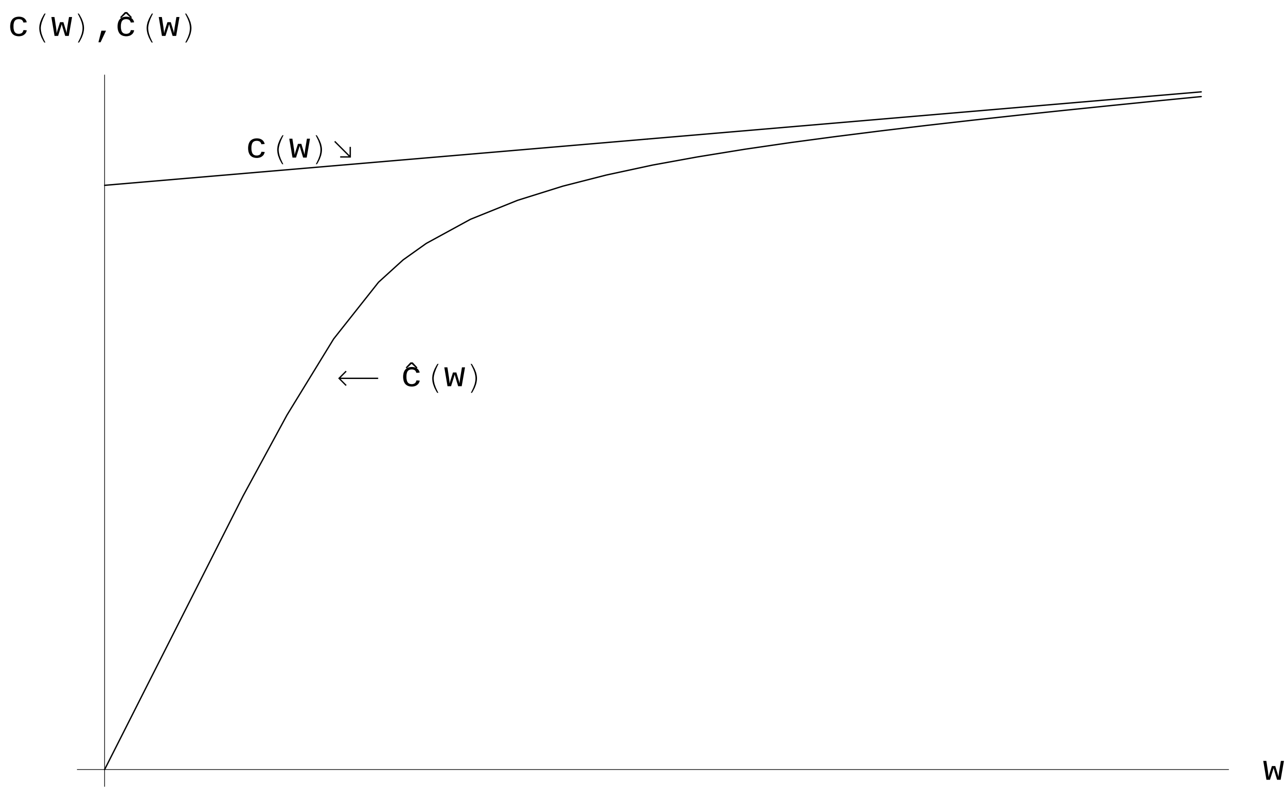
$$\mathbf{P}(\hat{c}(m))\hat{c}'(m) \;>\; \mathbf{P}(c(m))c'(m)$$

.

$\mathbf{P}(\hat{c}(m))\hat{c}^{\prime\prime}(m) > \mathbf{P}(c(m))c^{\prime\prime}(m)$?

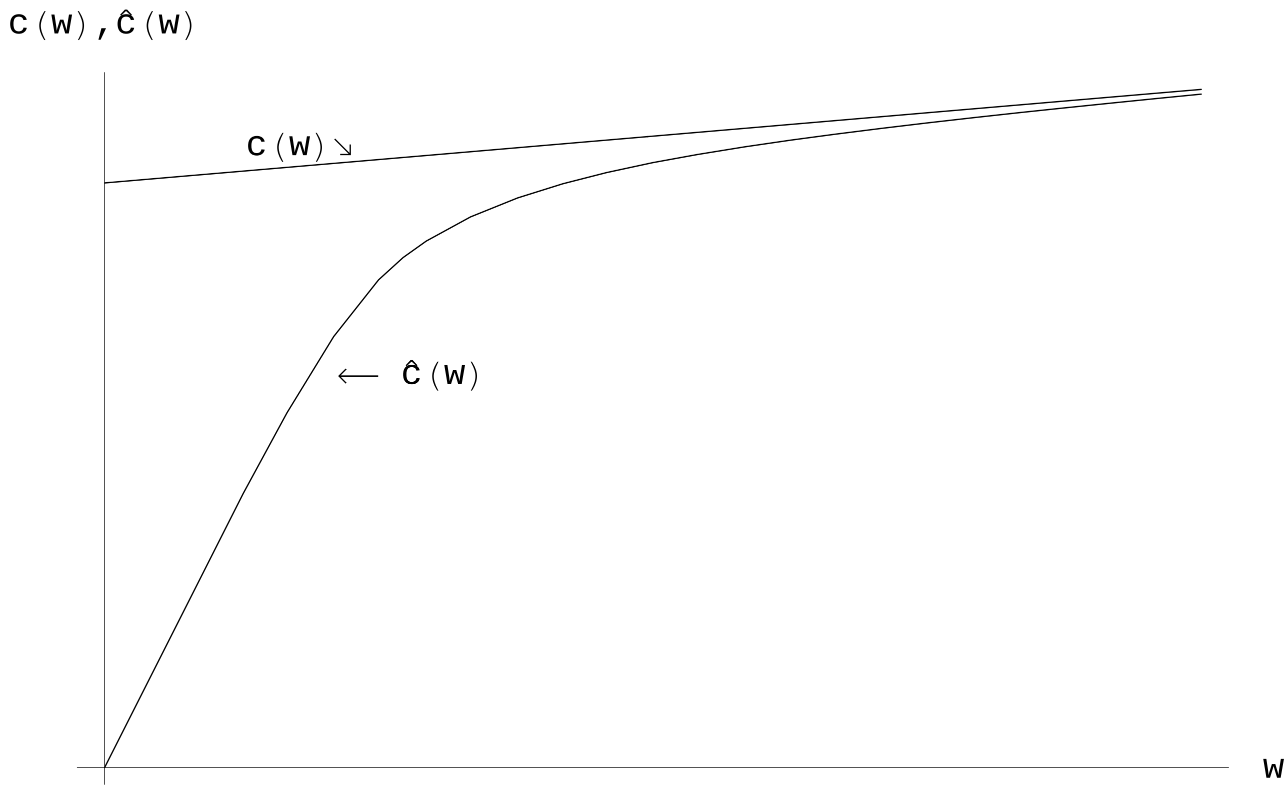


$\mathbf{P}(\hat{c}(m))\hat{c}^{\prime}(m) > \mathbf{P}(c(m))c^{\prime}(m)$?



Yes:

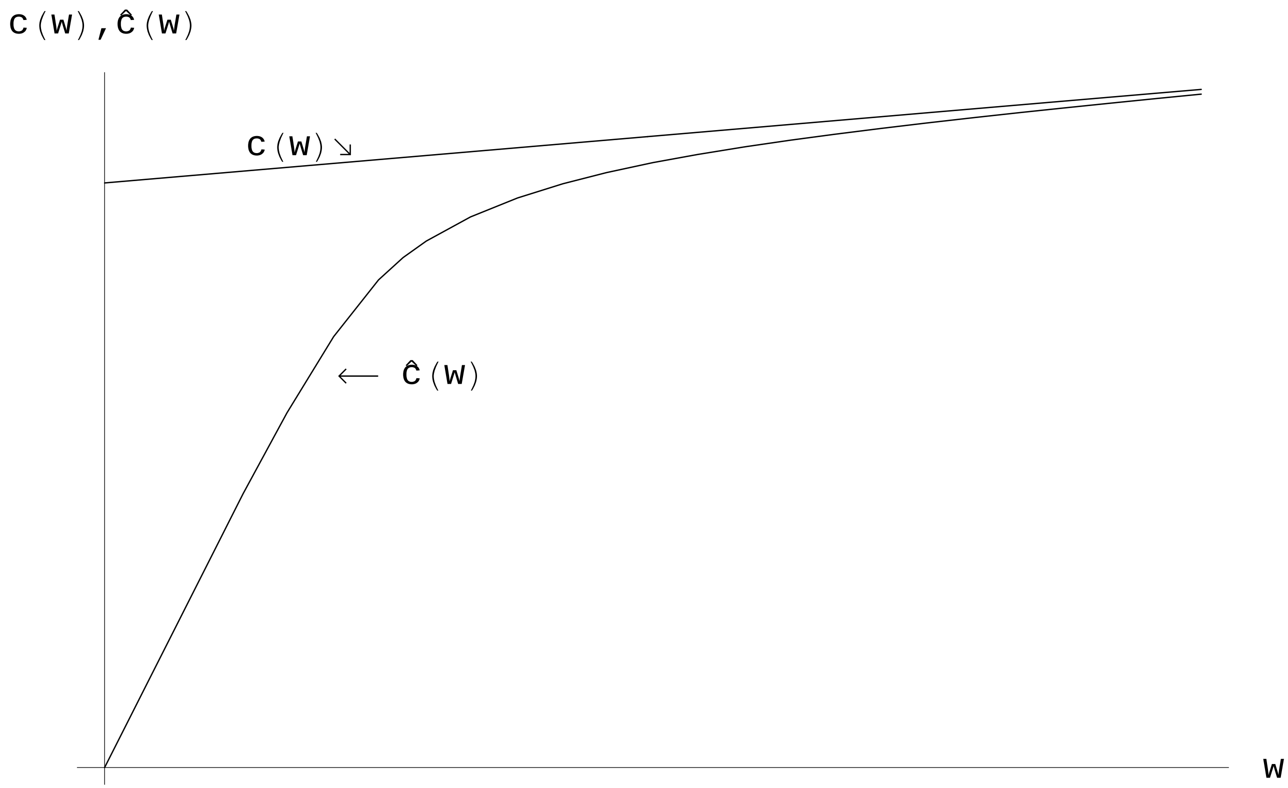
$\mathbf{P}(\hat{c}(m))\hat{c}^{\prime}(m) > \mathbf{P}(c(m))c^{\prime}(m)$?



Yes:

- $\hat{c} < c \Rightarrow \hat{\mathbf{P}} > \mathbf{P}$

$\mathbf{P}(\hat{c}(m))\hat{c}^{\square}(m) > \mathbf{P}(c(m))c^{\square}(m)$?



Yes:

- $\hat{c} < c$ $\square$ $\hat{\mathbf{P}} > \mathbf{P}$
- $\hat{c}' > c'$.

Imposing Constraints Backwards

Consider the consumption function in period $T-2$

Suppose:

Then:

Imposing Constraints Backwards

Consider the consumption function in period $T - 2$

Suppose:

- Life ends in $T + 1$ when $\mathbf{c}_{T+1}(m_{T+1}) = m_{T+1}$

Then:

Imposing Constraints Backwards

Consider the consumption function in period $T - 2$

Suppose:

- Life ends in $T + 1$ when $\mathbf{c}_{T+1}(m_{T+1}) = m_{T+1}$
- Intertemporally separable HARA utility

Then:

Imposing Constraints Backwards

Consider the consumption function in period $T - 2$

Suppose:

- Life ends in $T + 1$ when $\mathbf{c}_{T+1}(m_{T+1}) = m_{T+1}$
- Intertemporally separable HARA utility
- Baseline situation with no constraints

Then:

Imposing Constraints Backwards

Consider the consumption function in period $T - 2$

Suppose:

- Life ends in $T + 1$ when $\mathbf{c}_{T+1}(m_{T+1}) = m_{T+1}$
- Intertemporally separable HARA utility
- Baseline situation with no constraints

Then:

- First impose constraint in T □ $\mathbf{c}_{T-2,T}$

Imposing Constraints Backwards

Consider the consumption function in period $T - 2$

Suppose:

- Life ends in $T + 1$ when $\mathbf{c}_{T+1}(m_{T+1}) = m_{T+1}$
- Intertemporally separable HARA utility
- Baseline situation with no constraints

Then:

- First impose constraint in T □ $\mathbf{c}_{T-2,T}$
- Then impose constraint in $T - 1$ □ $\mathbf{c}_{T-2,T-1}$

Imposing Constraints Backwards

Consider the consumption function in period $T - 2$

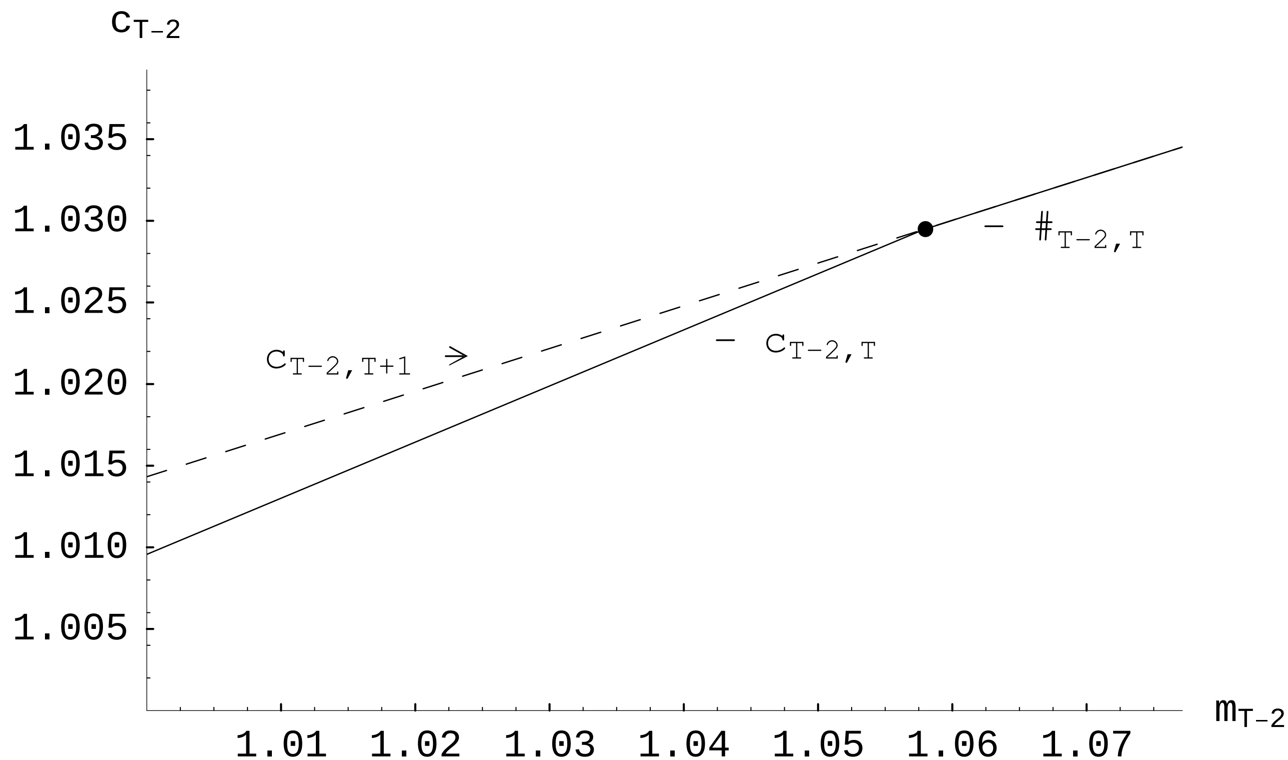
Suppose:

- Life ends in $T + 1$ when $\mathbf{c}_{T+1}(m_{T+1}) = m_{T+1}$
- Intertemporally separable HARA utility
- Baseline situation with no constraints

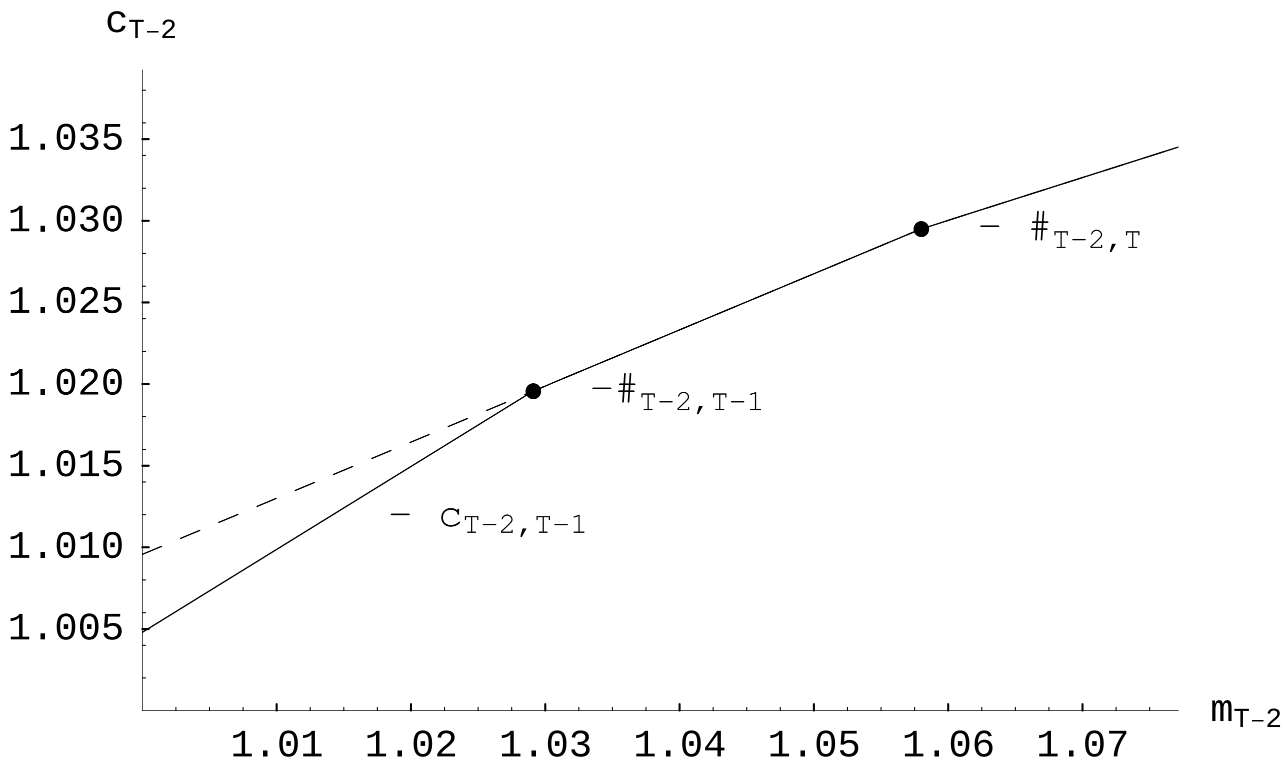
Then:

- First impose constraint in T □ $\mathbf{c}_{T-2,T}$
- Then impose constraint in $T - 1$ □ $\mathbf{c}_{T-2,T-1}$
- Then impose constraint in $T - 2$ □ $\mathbf{c}_{T-2,T-2}$.

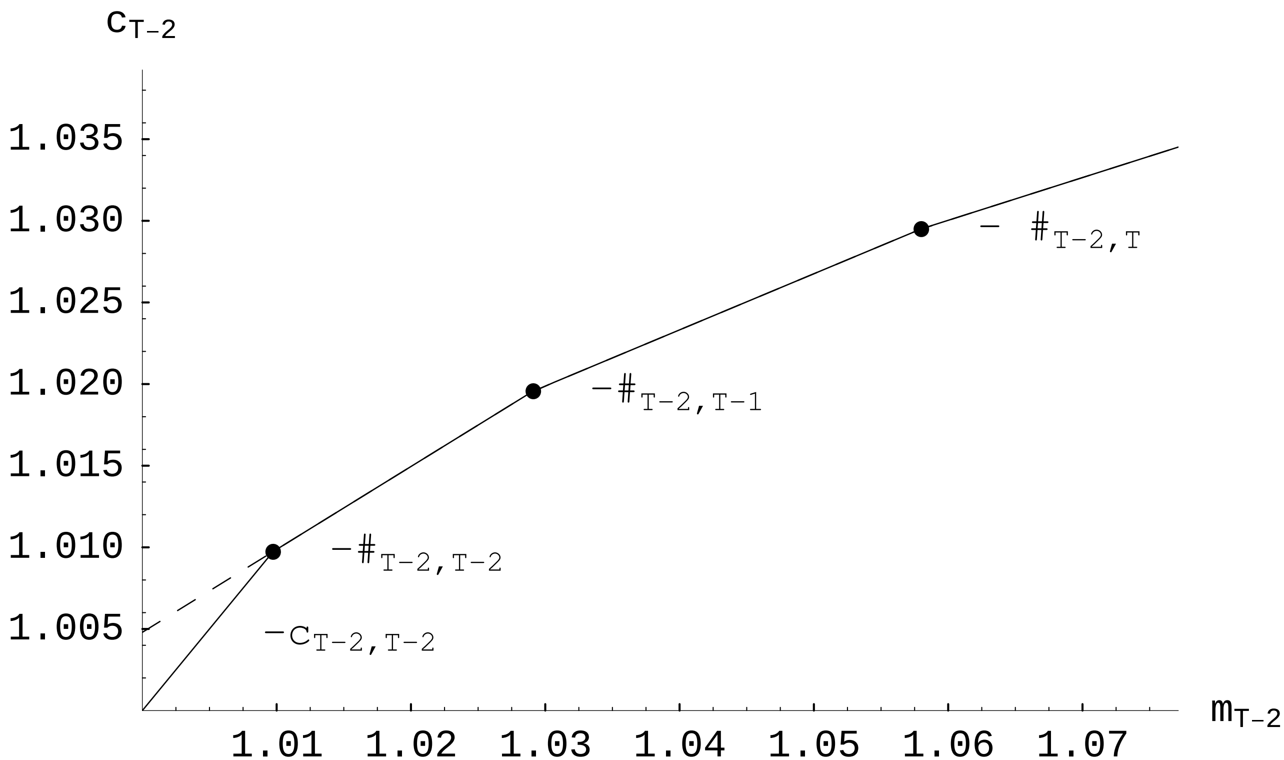
Counterclockwise Concavification



Counterclockwise Concavification



Counterclockwise Concavification



Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

- For $m \geq m$:

Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

- For $m \geq m$:
  - $\hat{c}(m) = c(m)$

Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

- For $m \geq m$:
  - $\hat{c}(m) = c(m)$
- For $m < m$:

Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

- For $m \geq m$:
  - $\hat{c}(m) = c(m)$
- For $m < m$:
  - $\hat{c}'(m) > c'(m)$.

Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

- For $m \geq m$:
  - $\hat{c}(m) = c(m)$
- For $m < m$:
  - $\hat{c}'(m) > c'(m)$.

P $(\hat{c}(m\ ))\hat{c}'(m\ )$ undefined because $\hat{c}''(m\ ) = \infty$

Counterclockwise Concavification

$\hat{c}(m)$ is a CCC of $c(m)$ around m if $\hat{c}(m)$ continuous and

- For m ≥ m :
  - $\hat{c}(m) = c(m)$
- For $m < m$:
  - $\hat{c}'(m) > c'(m)$.

P$(\hat{c}(m\ \ ))\hat{c}'(m\ \ )$ undefined because $\hat{c}''(m\ \ ) =$ **∞**

☐ Think of prudence as infinite at kink points .

C C C Increases Prudence

C C C Increases Prudence

- Key theorem:

C C C Increases Prudence

- Key theorem:
  - If $\hat{c}(m)$ is a CCC of $c(m)$, then going from $c(m)$ to $\hat{c}(m)$ increases prudence at all m

C C C Increases Prudence

- Key theorem:
  - If $\hat{c}(m)$ is a CCC of $c(m)$, then going from $c(m)$ to $\hat{c}(m)$ increases prudence at all m
- Application:
  - $\mathbf{c}_{T-n,T-(q+1}(m)$ is a CCC of $\mathbf{c}_{T-n,T-q}(m)$ for $q < n$

C C C Increases Prudence

- Key theorem:
  - If $\hat{c}(m)$ is a CCC of $c(m)$, then going from $c(m)$ to $\hat{c}(m)$ increases prudence at all m
- Application:
  - $\mathbf{c}_{T-n,T-(q+1}(m)$ is a CCC of $\mathbf{c}_{T-n,T-q}(m)$ for $q < n$
  - $\mathbf{c}_{T-n,T-(q+1}(m)$ is 'globally more prudent'

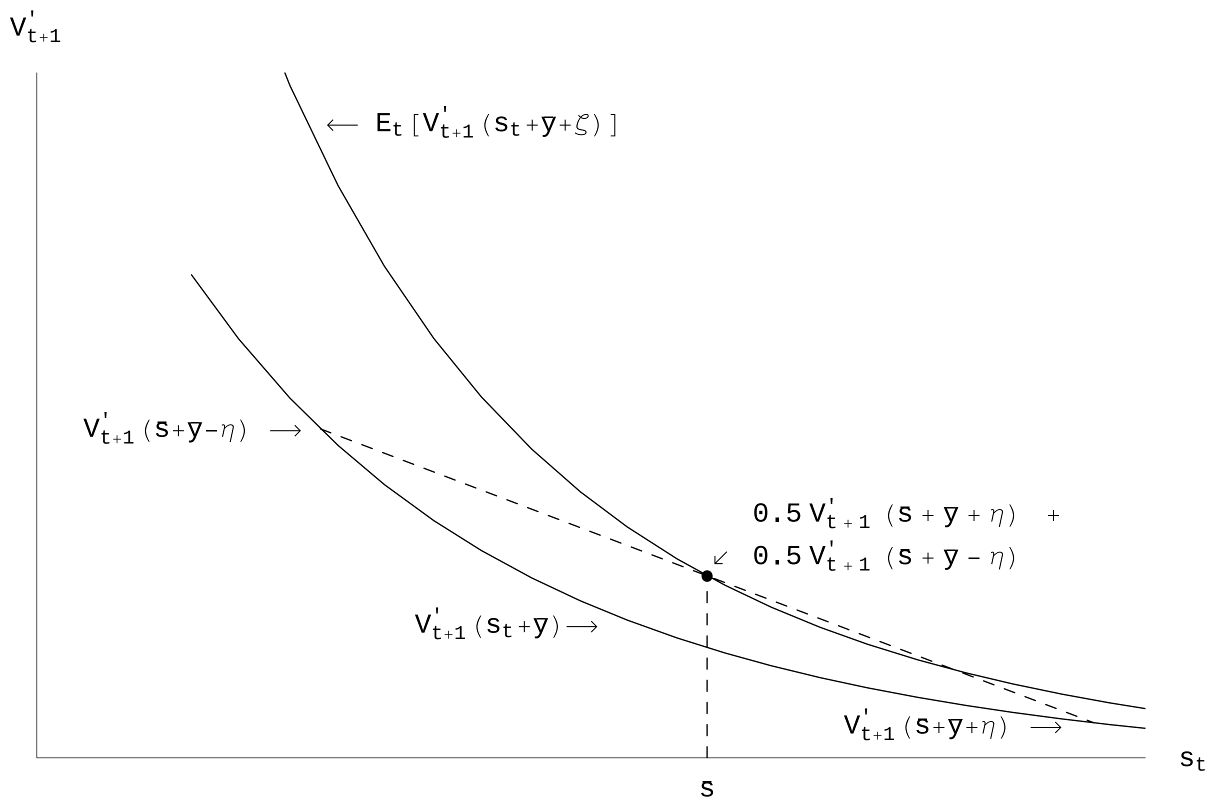
C C C Increases Prudence

- Key theorem:
  - If $\hat{c}(m)$ is a CCC of $c(m)$, then going from $c(m)$ to $\hat{c}(m)$ increases prudence at all m
- Application:
  - $\mathbf{c}_{T-n,T-(q+1)}(m)$ is a CCC of $\mathbf{c}_{T-n,T-q}(m)$ for $q < n$
  - $\mathbf{c}_{T-n,T-(q+1)}(m)$ is 'globally more prudent'

That is, imposing each earlier constraint increases the prudence of the consumption function

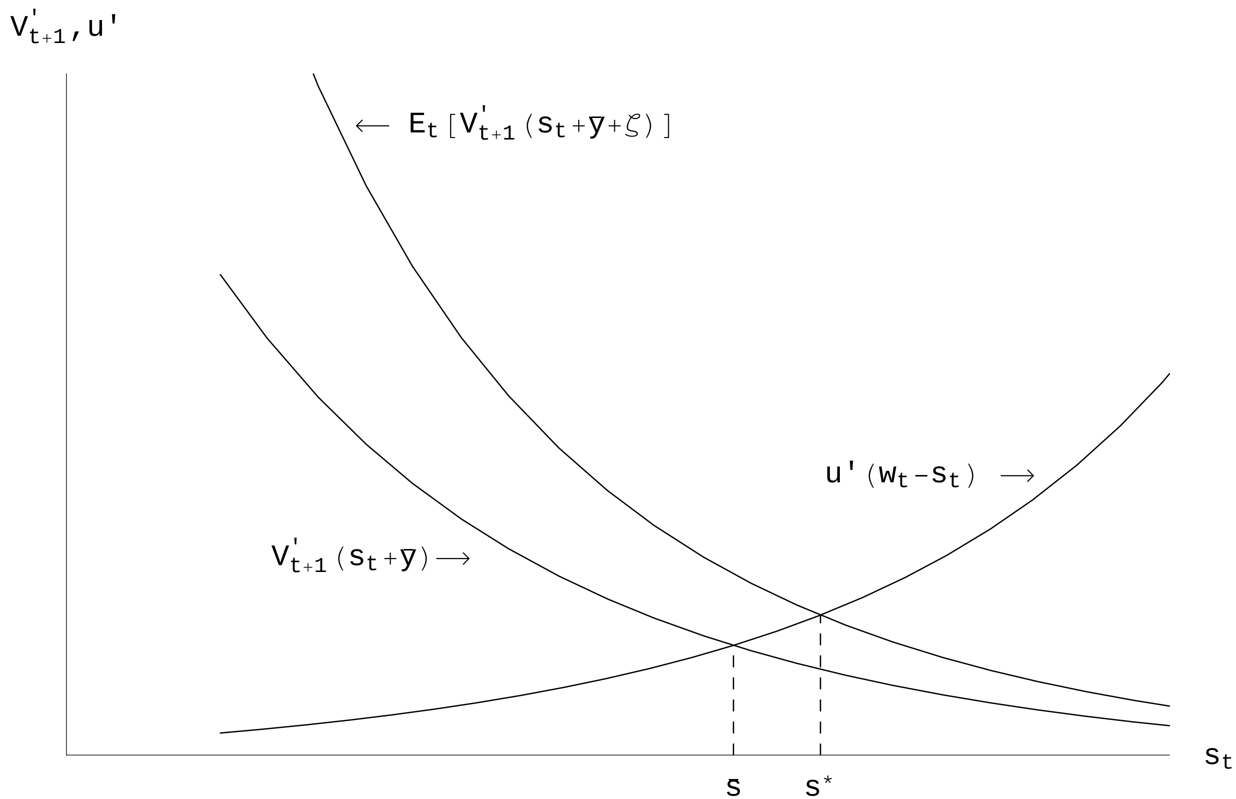
.

Precautionary Saving



Symmetric Two Point Background Risk

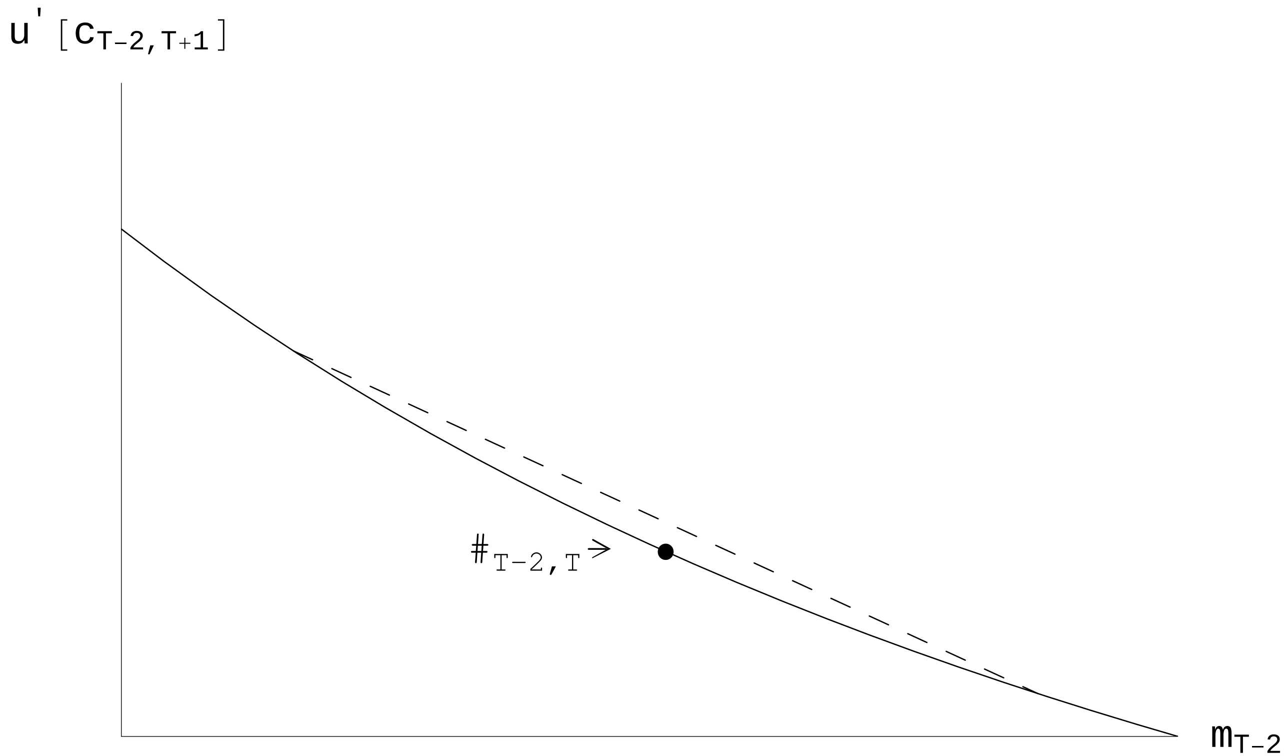
Finding Optimal Saving



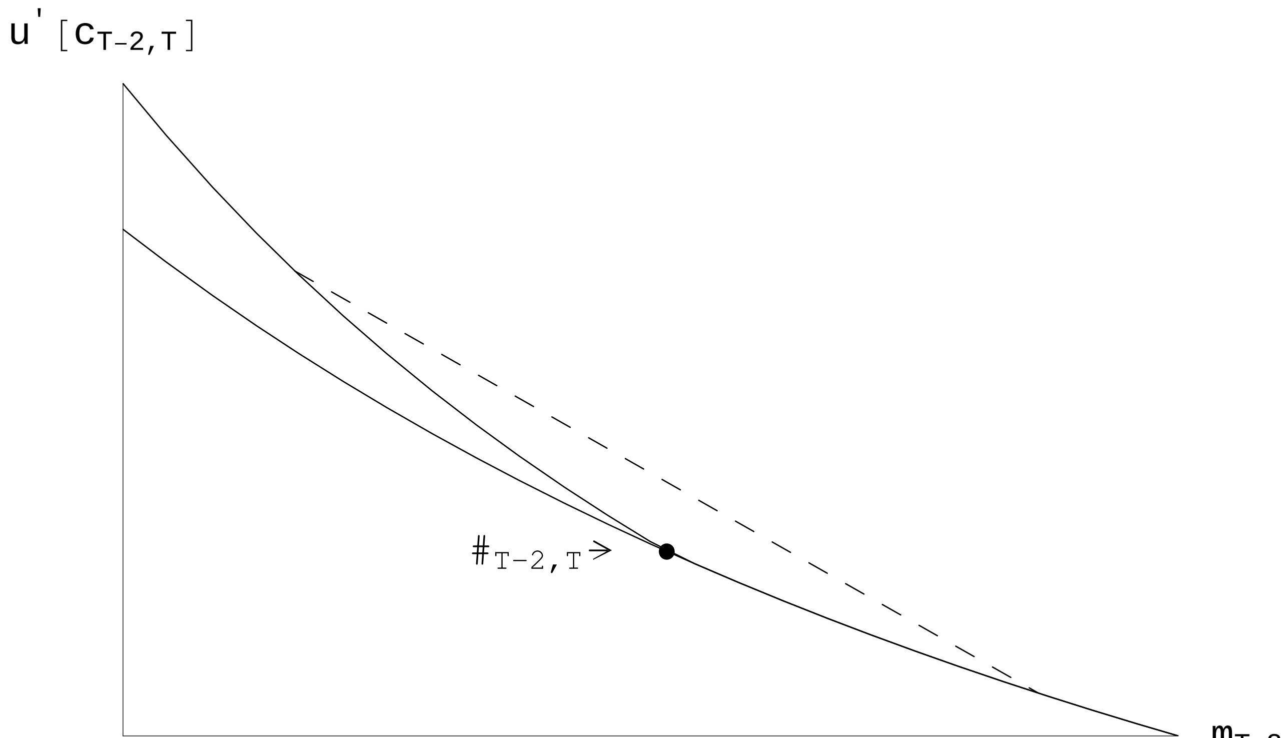
$$u'(c_t) = E_t[V'_{t+1}(m_{t+1})]$$

.

C C C Effect



C C C Effect



But ...

Given a baseline $c(m)$ that is *not* linear (perhaps because of some initial constraints),

But ...

Given a baseline $c(m)$ that is *not* linear (perhaps because of some initial constraints),

imposing a *new* constraint that will hold at some date in the future will probably *not* generate a $\hat{c}(m)$ that is a CCC of $c(m)$

.

Example

Setup: Impatient consumer with

Example

Setup: Impatient consumer with

- 3 period life, t to $t+2=T+1$

Example

Setup: Impatient consumer with

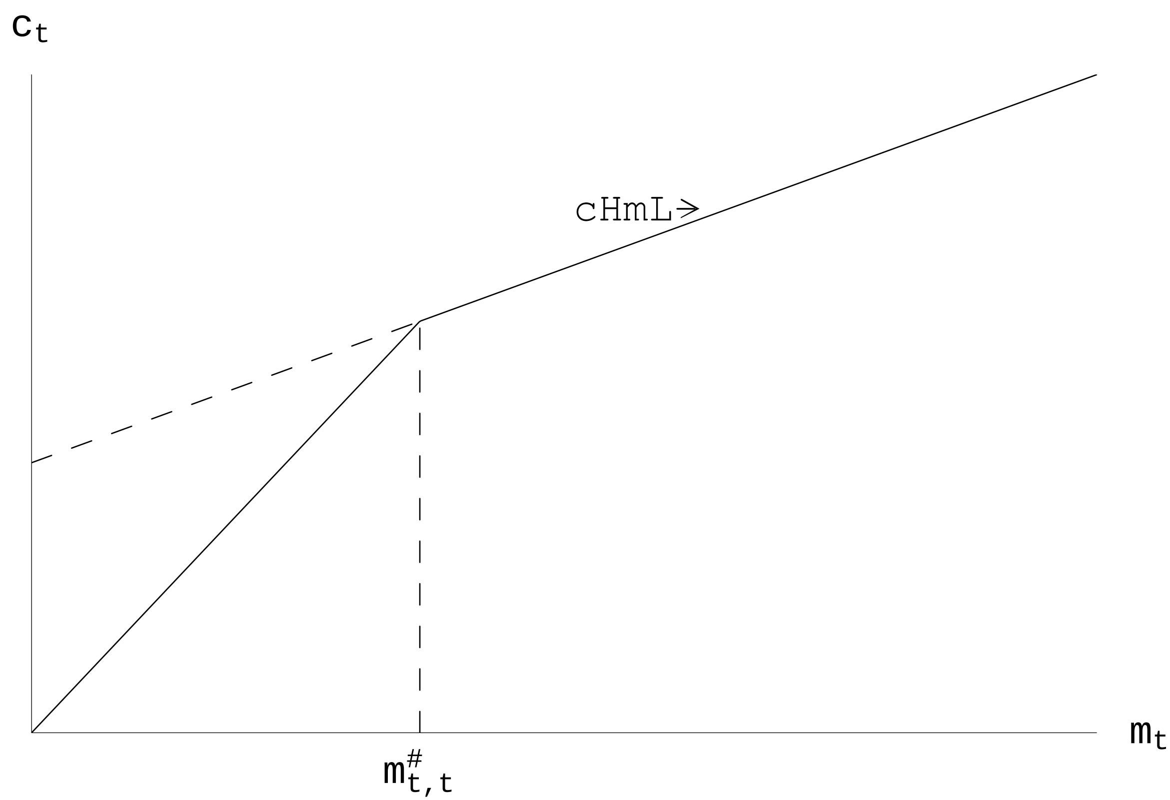
- 3 period life, t to $t + 2 = T + 1$
- Social Security income = 1 in period $t + 2$

Example

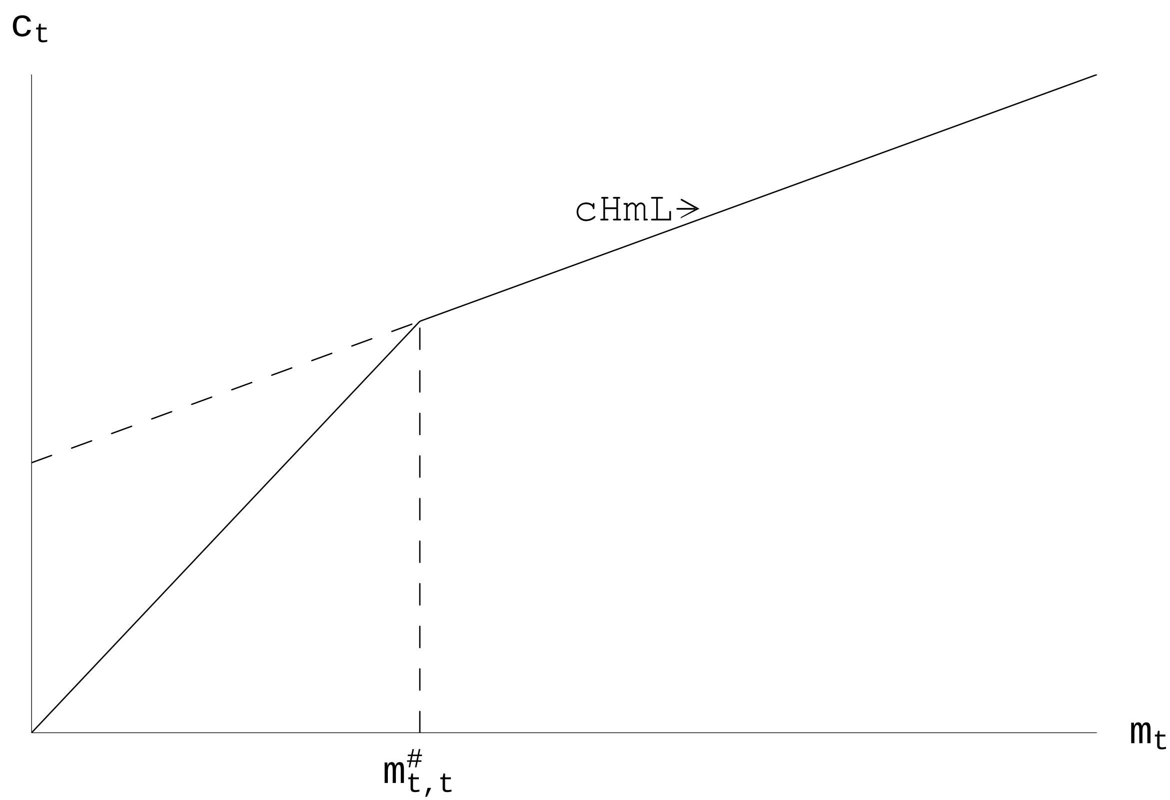
Setup: Impatient consumer with

- 3 period life, t to $t+2=T+1$
- Social Security income = 1 in period $t+2$
- Labor income = 1 in periods t and $t+1$.

Baseline Constraints: T_t

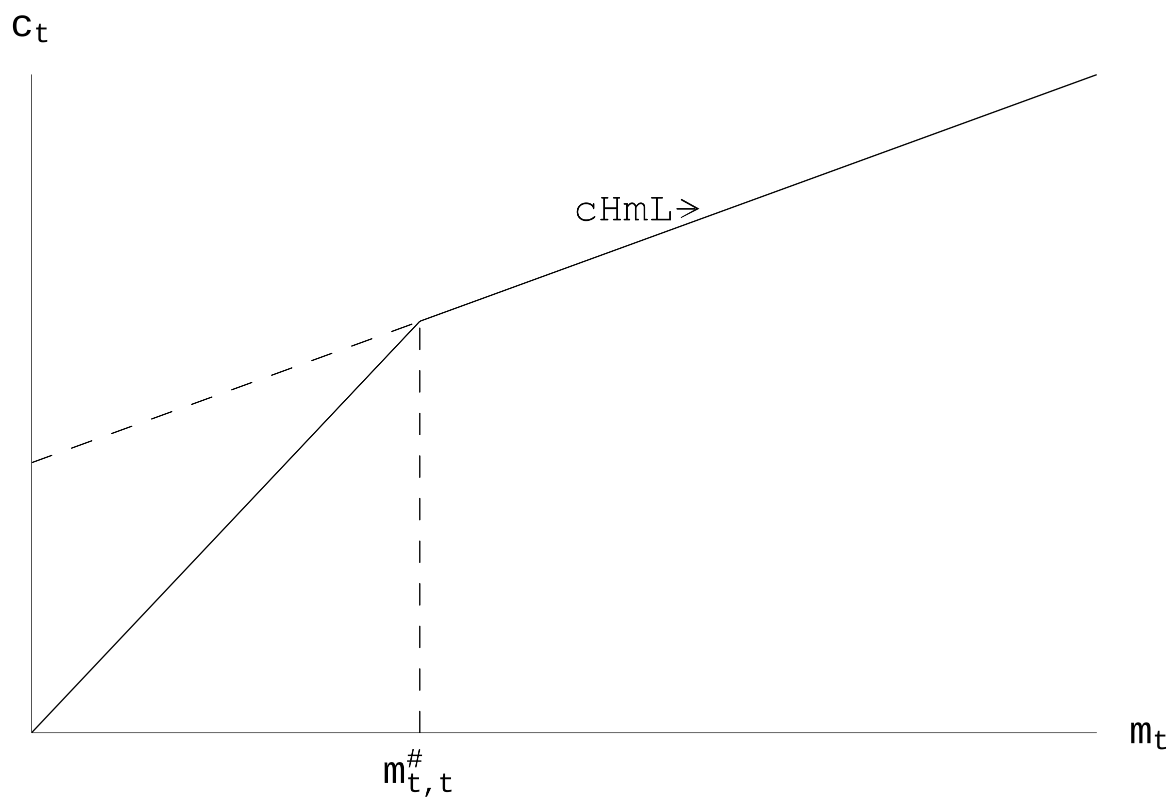


Baseline Constraints: $\mathbf{T}_t$



- Baseline: Constraint *only* at date t (constraint set $\mathbf{T}_t$) Liquidity Co

Baseline Constraints: $\mathbf{T}_t$



- Baseline: Constraint *only* at date t (constraint set $\mathbf{T}_t$)
- Induces kink in $\mathbf{c}_{t,t}(m)$ at $m_{t,1}$

$$\hat{\mathbf{T}}_t = \square\{\mathbf{T}_t, c_{t+1} \geq m_{t+1}\}$$

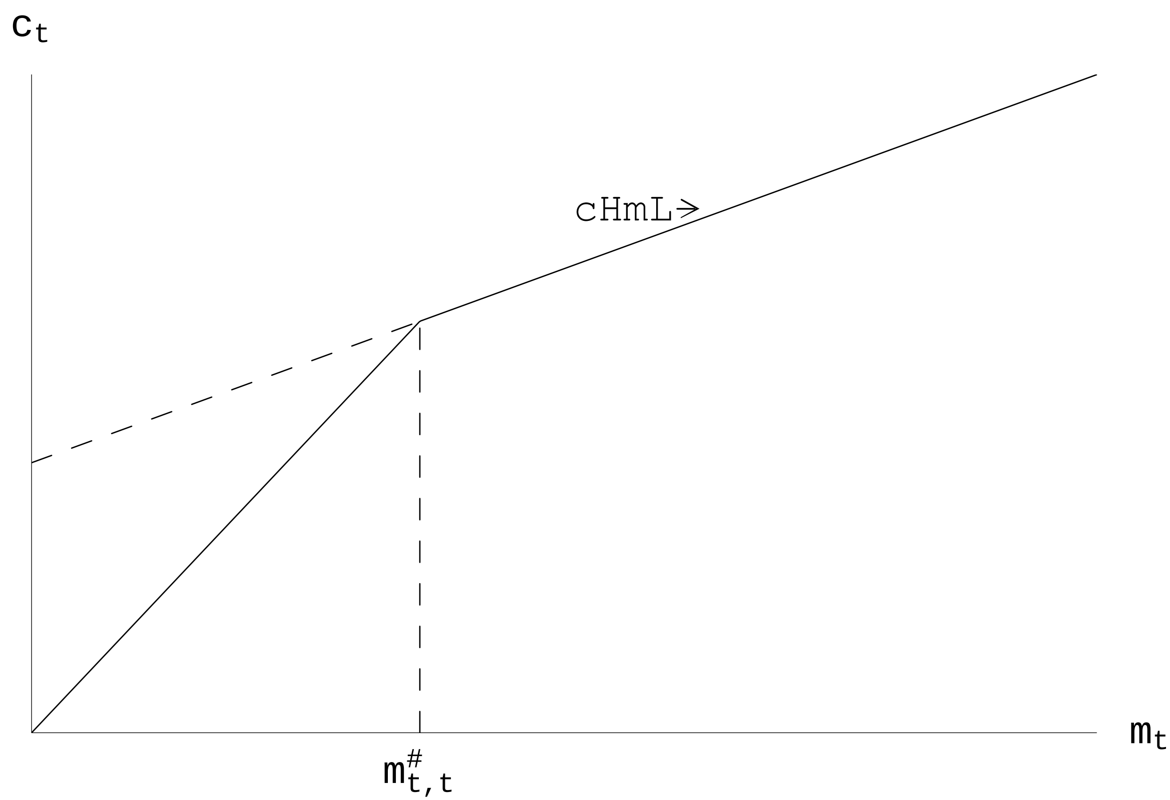
$$\hat{\mathbf{T}}_t = \square\{\mathbf{T}_t, c_{t+1} \geq m_{t+1}\}$$

- Can’t borrow against SS

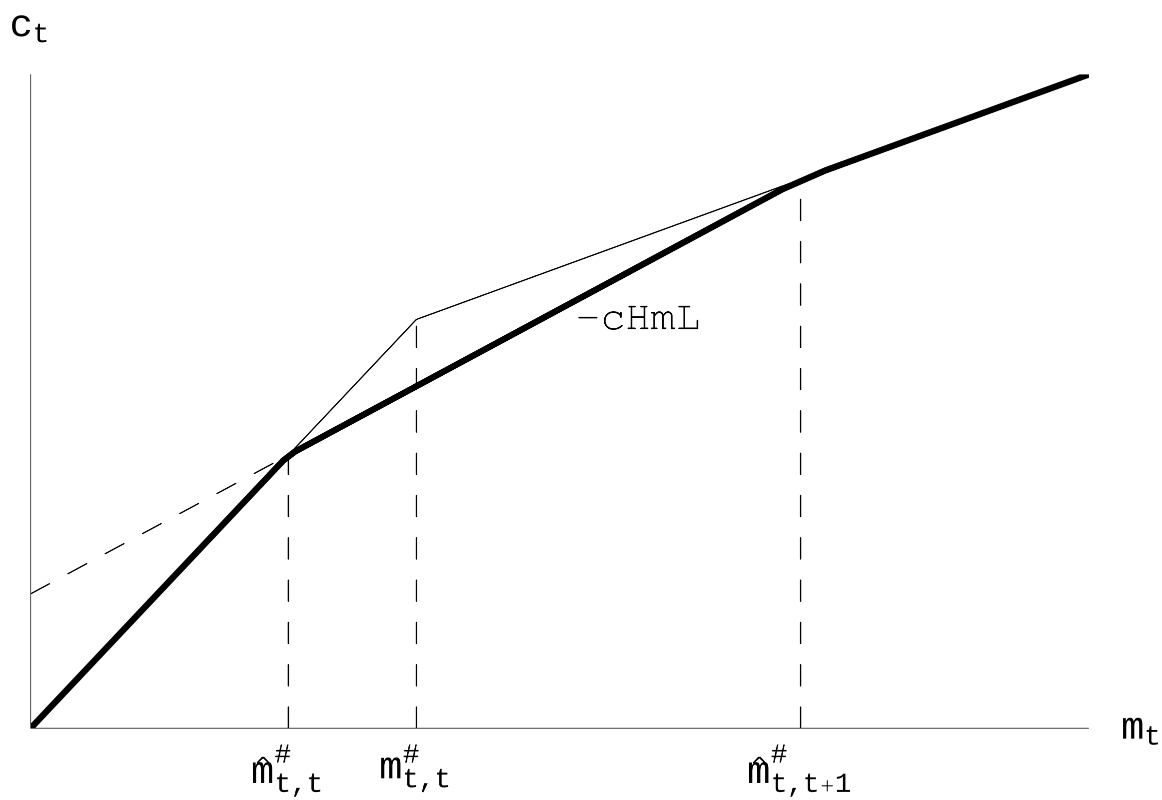
$\hat{\mathbf{T}}_t = \square\{\mathbf{T}_t, c_{t+1} \geq m_{t+1}\}$

- Can't borrow against SS
- *Want* to *plan* to borrow against SS if $\hat{m}_{t,t} < m_t < \hat{m}_{t,t+1}$.

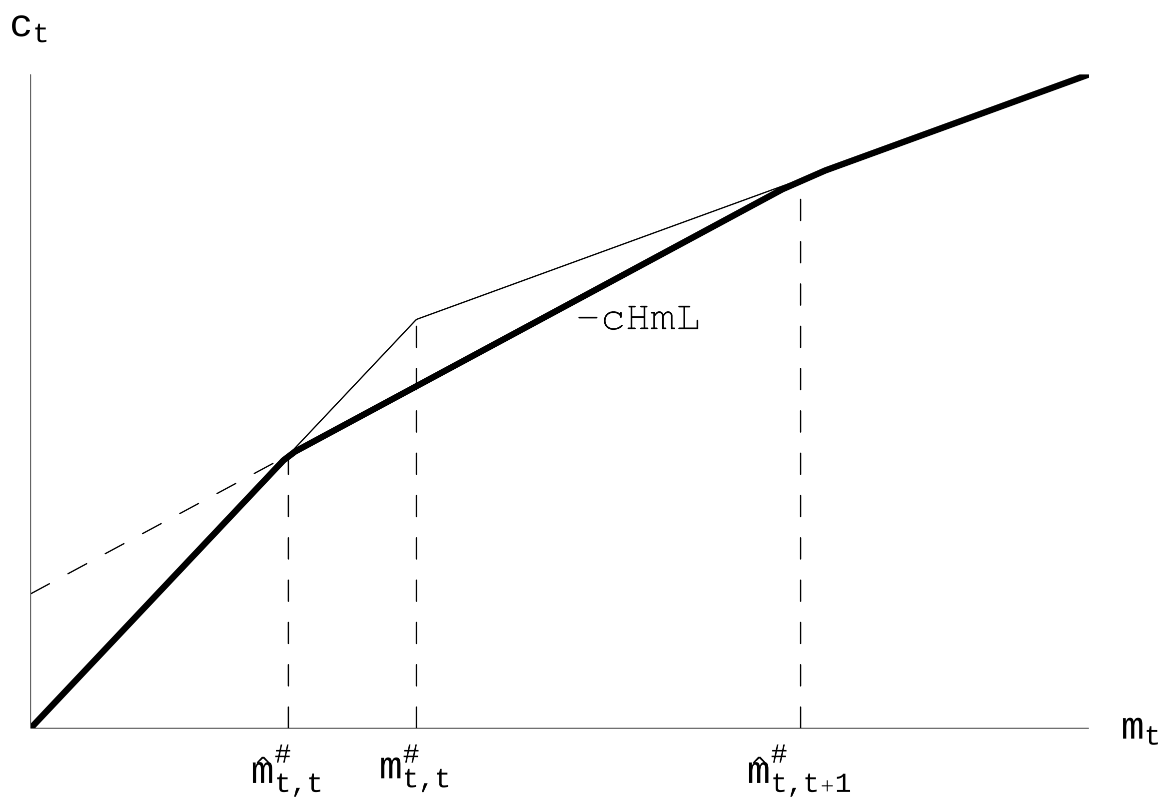
Graphically



Graphically

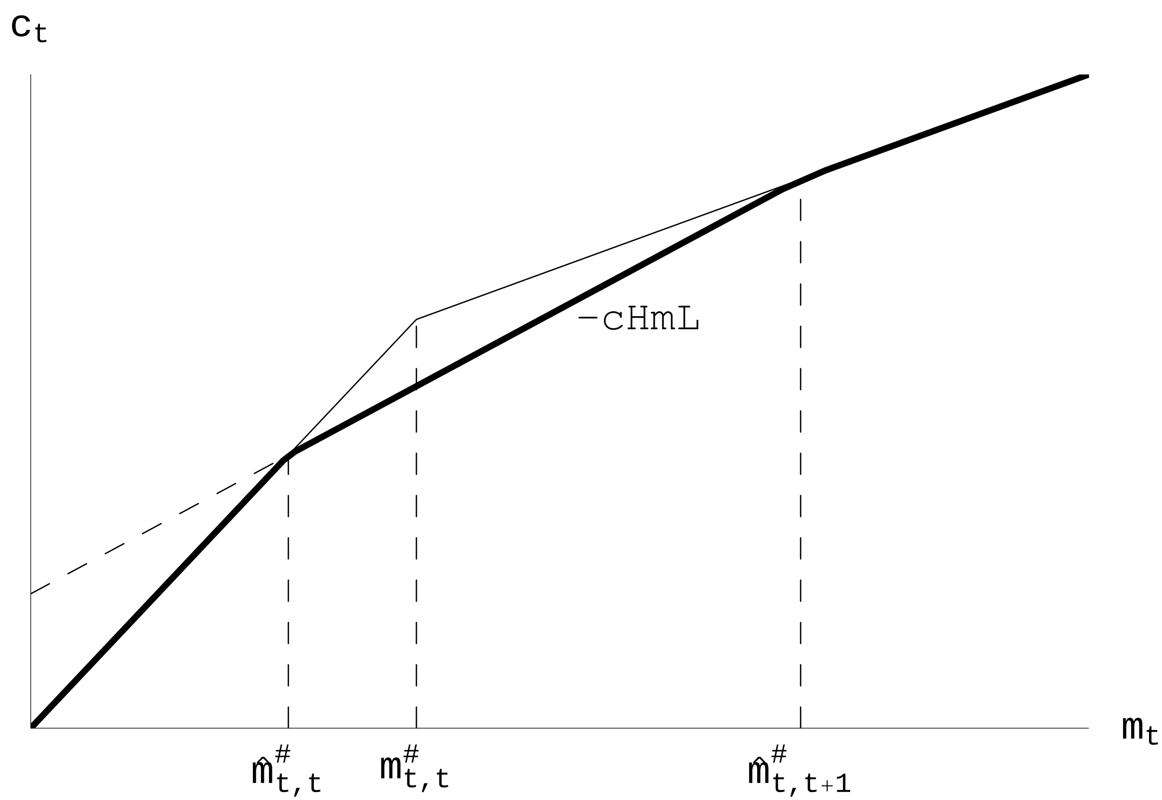


Graphically



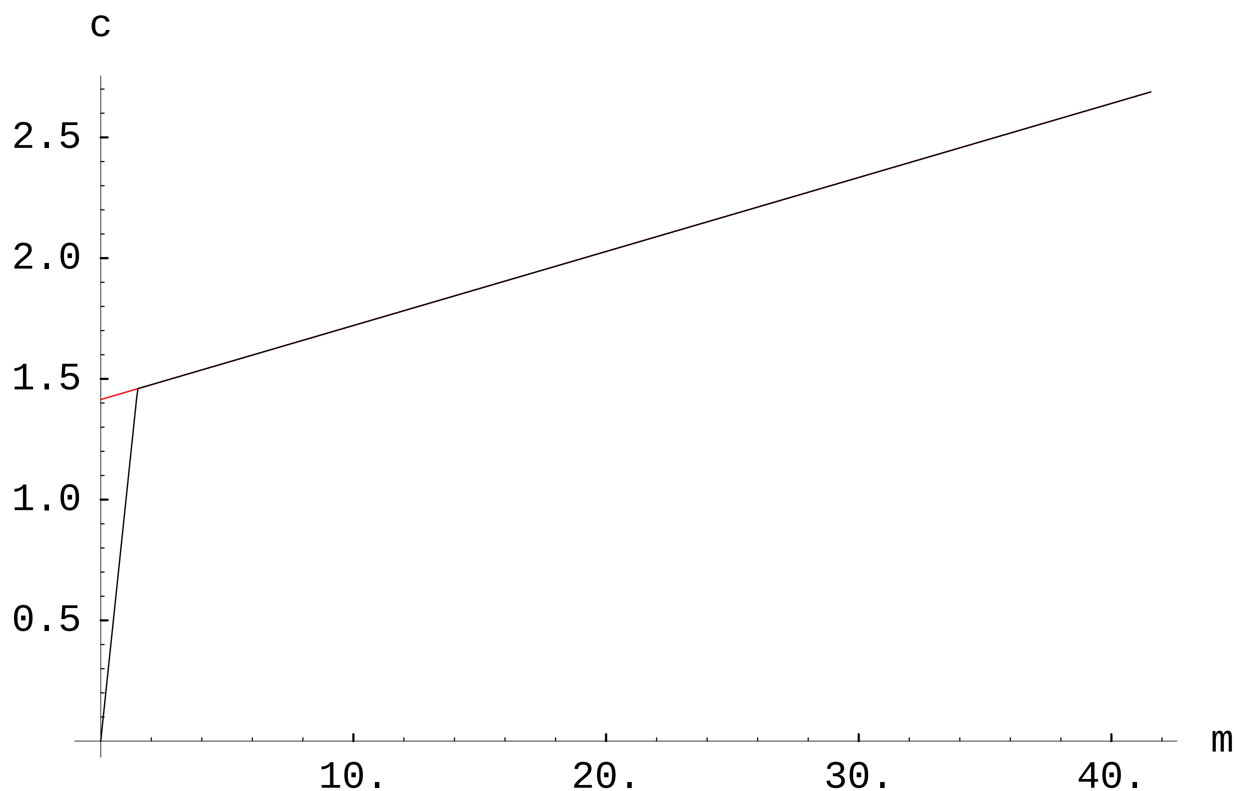
- Baseline: $c(m)$ prudence **∞** at $m_{t,t}$

Graphically

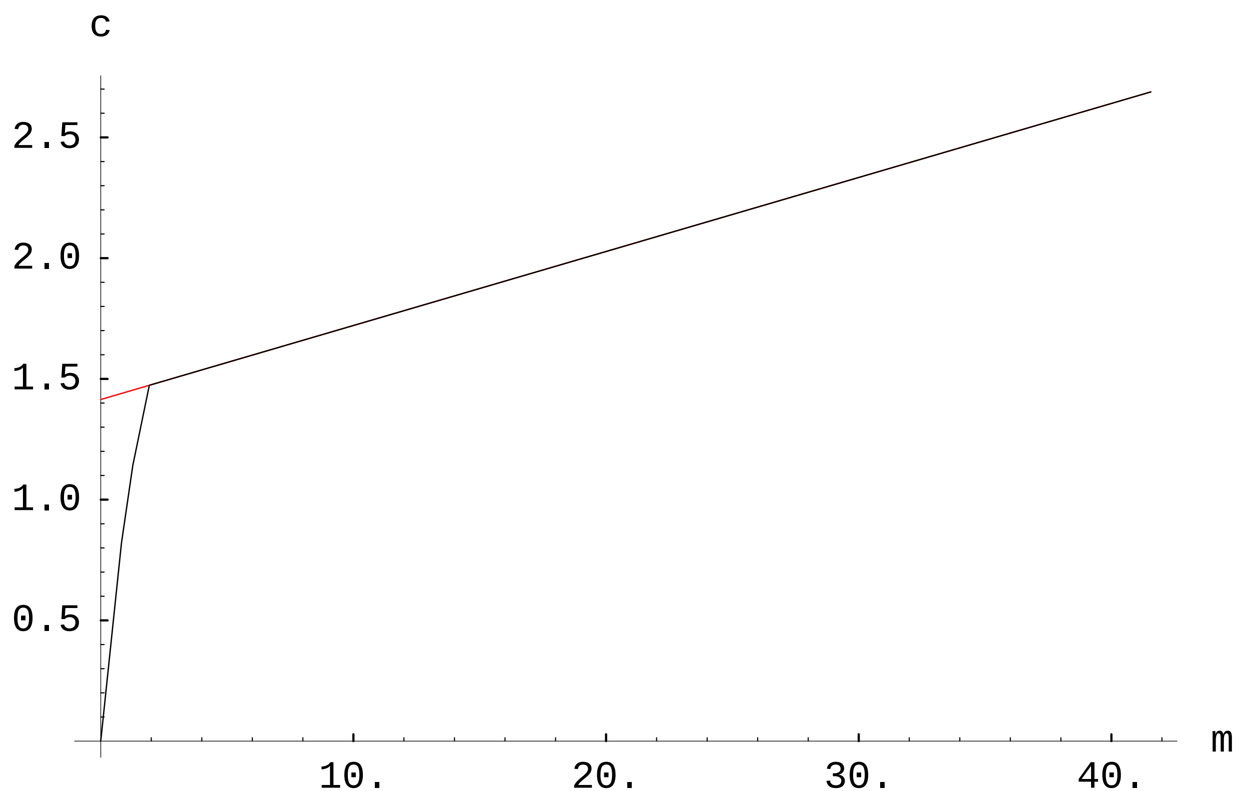


- Baseline: $c(m)$ prudence ∞ at $m_{t,t}$
- Modified: $\hat{c}(m)$ prudence finite at $m_{t,t}$

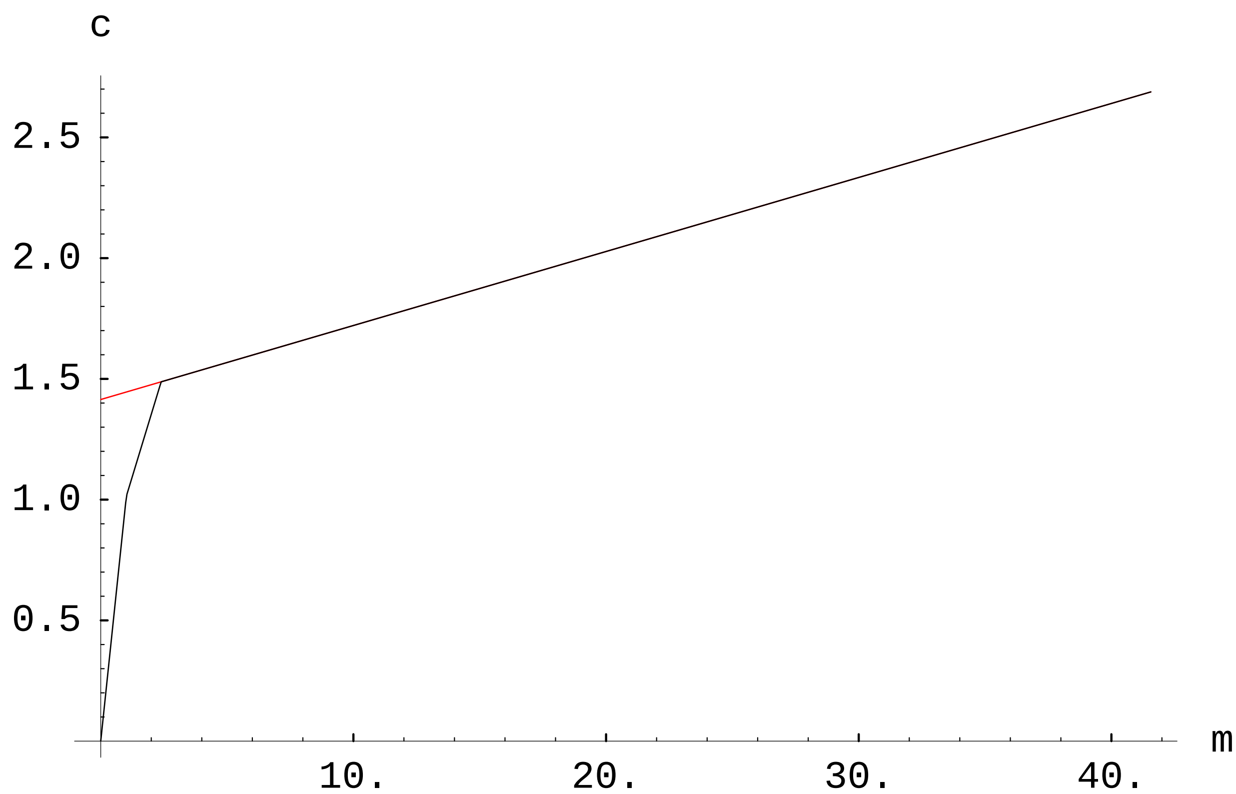
Imposing Constraints Forward



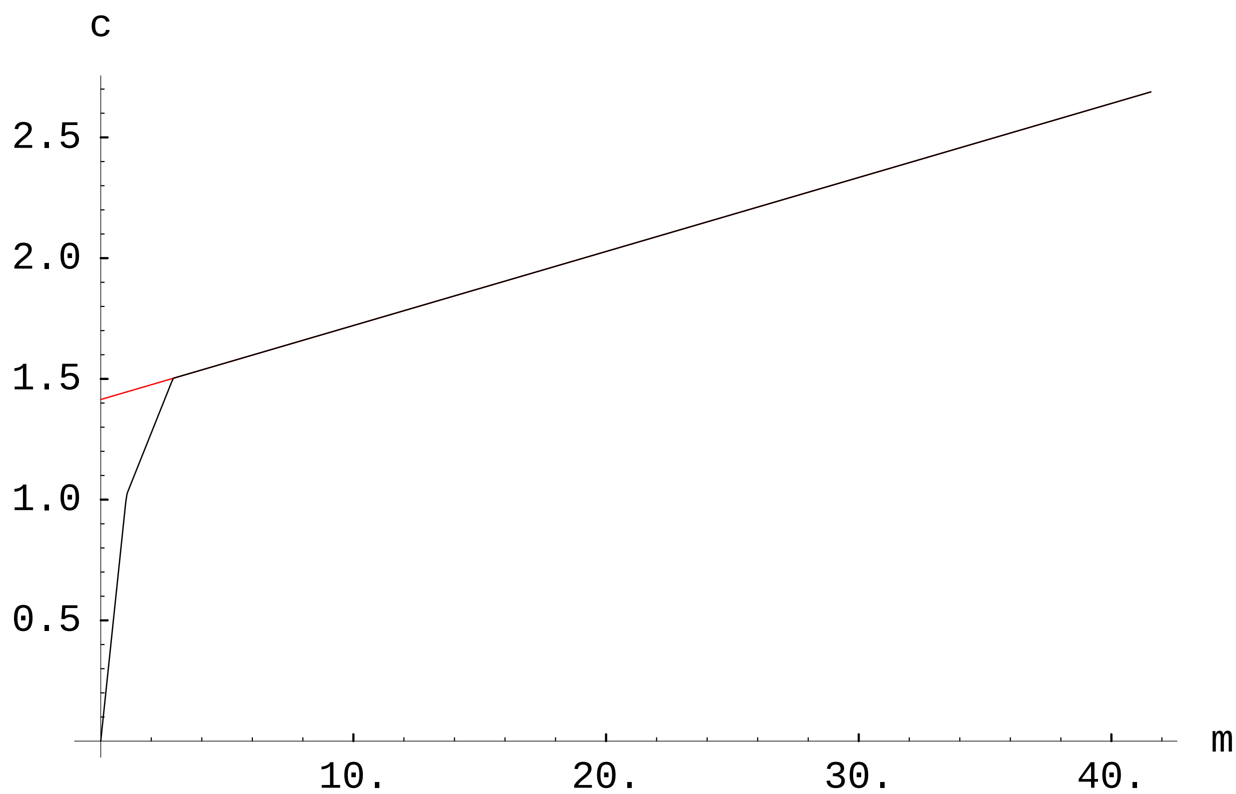
Imposing Constraints Forward



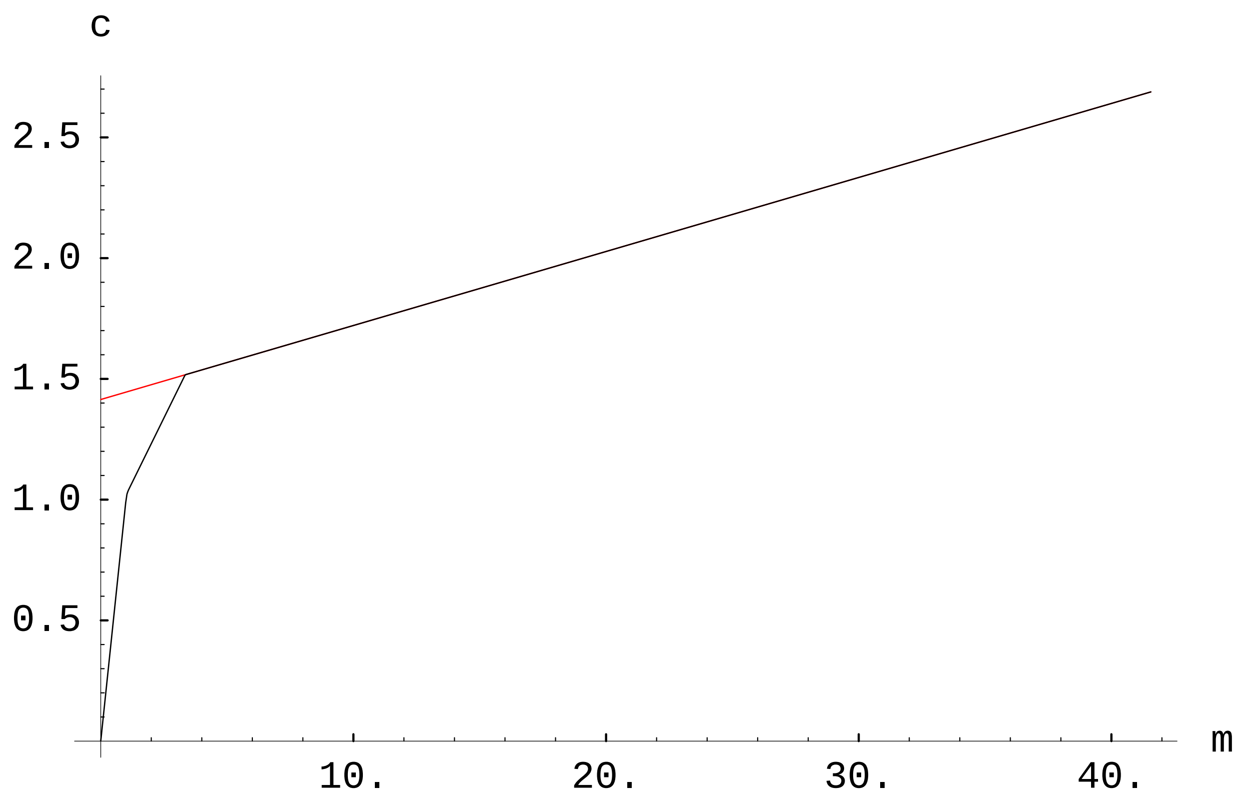
Imposing Constraints Forward



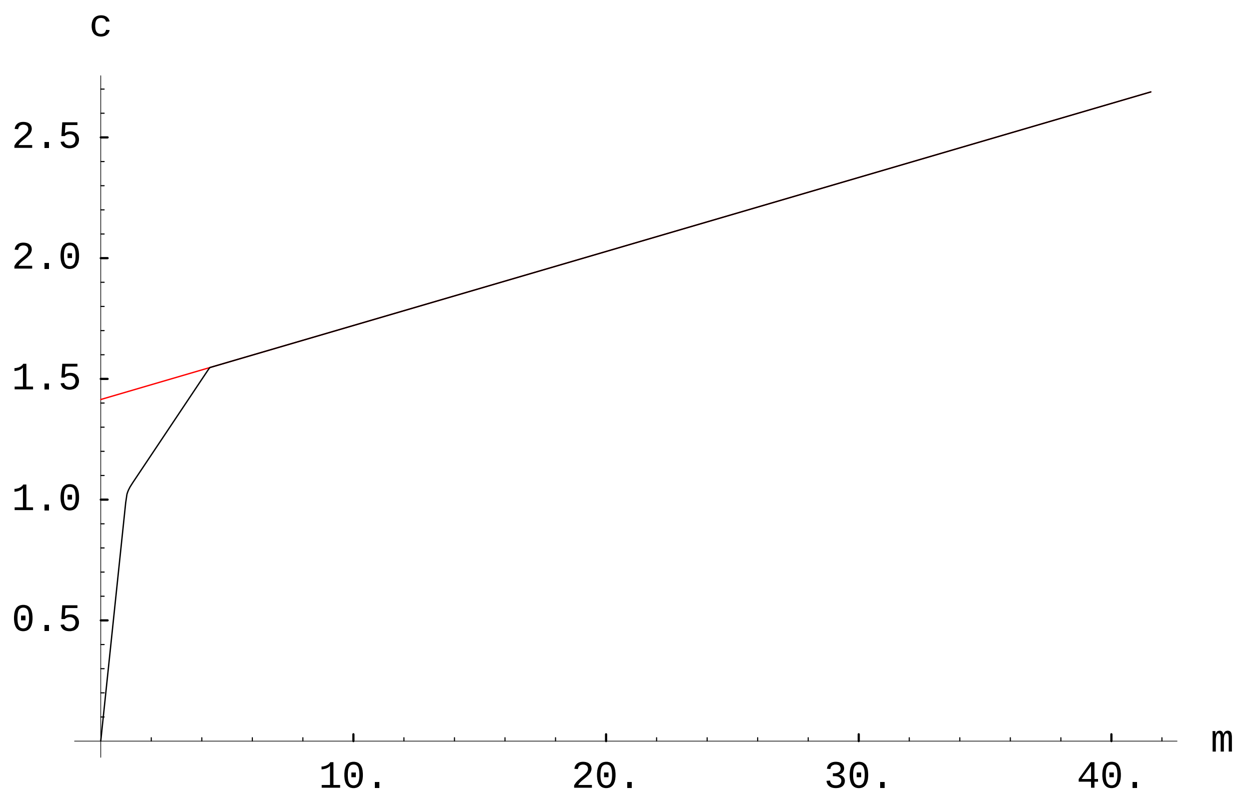
Imposing Constraints Forward



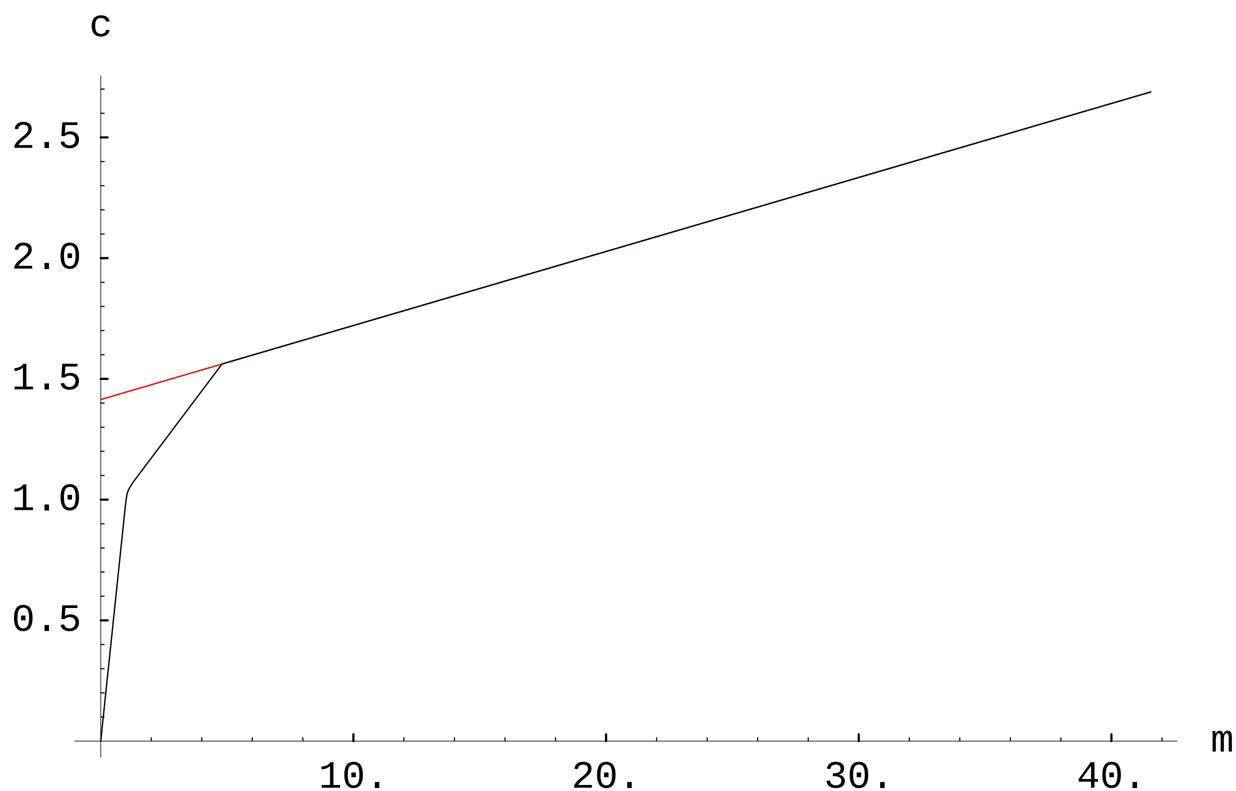
Imposing Constraints Forward



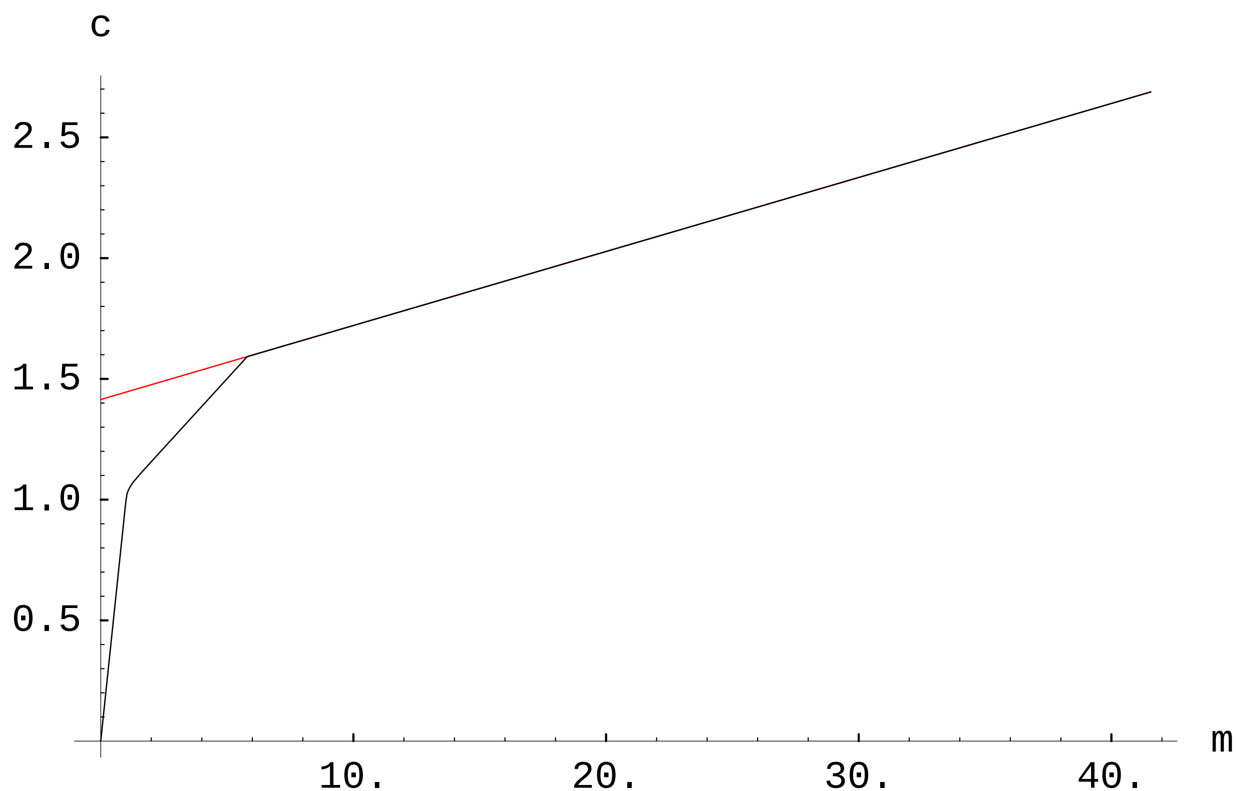
Imposing Constraints Forward



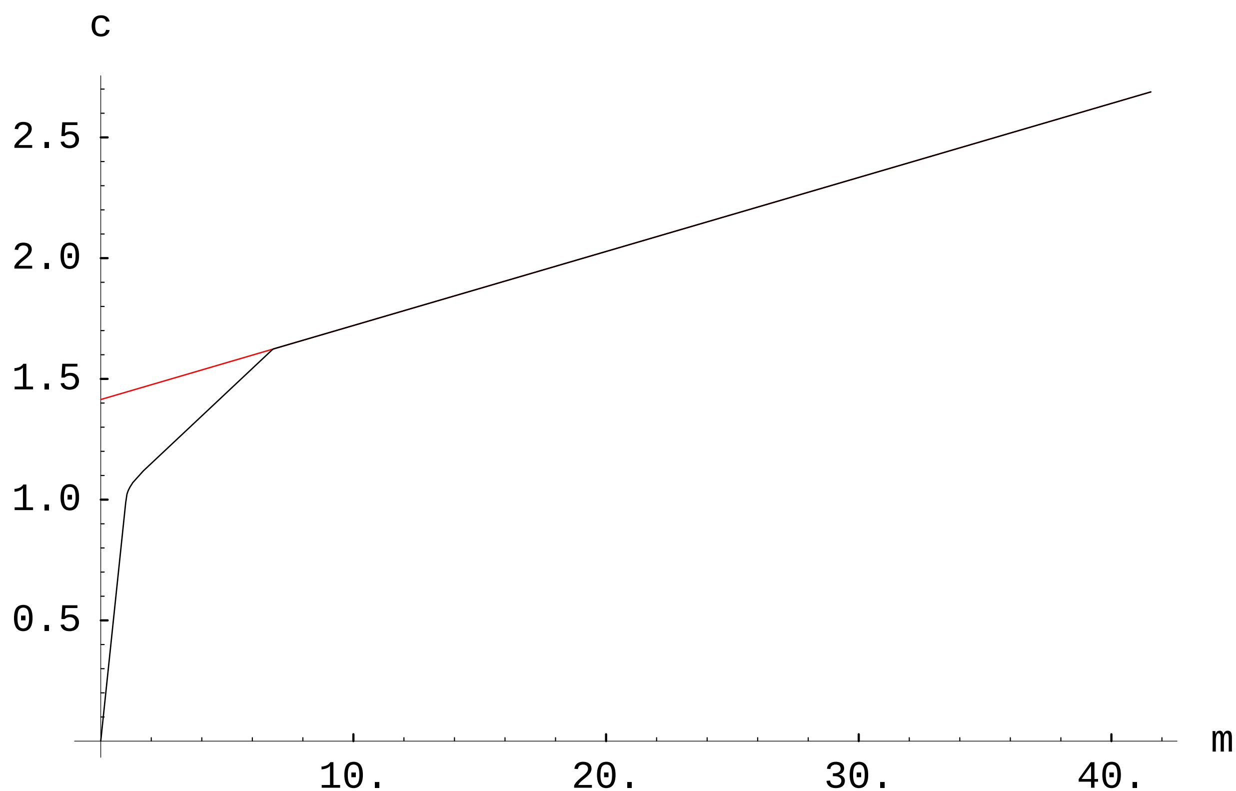
Imposing Constraints Forward



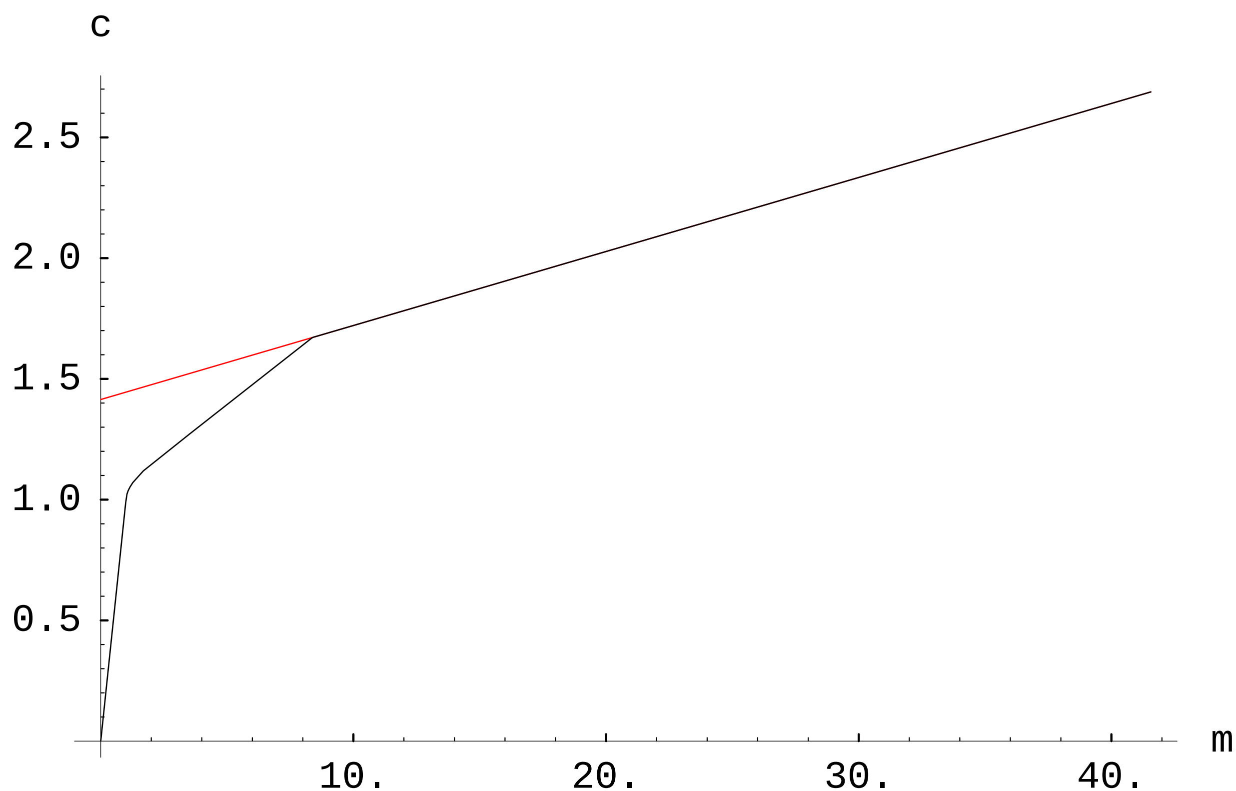
Imposing Constraints Forward



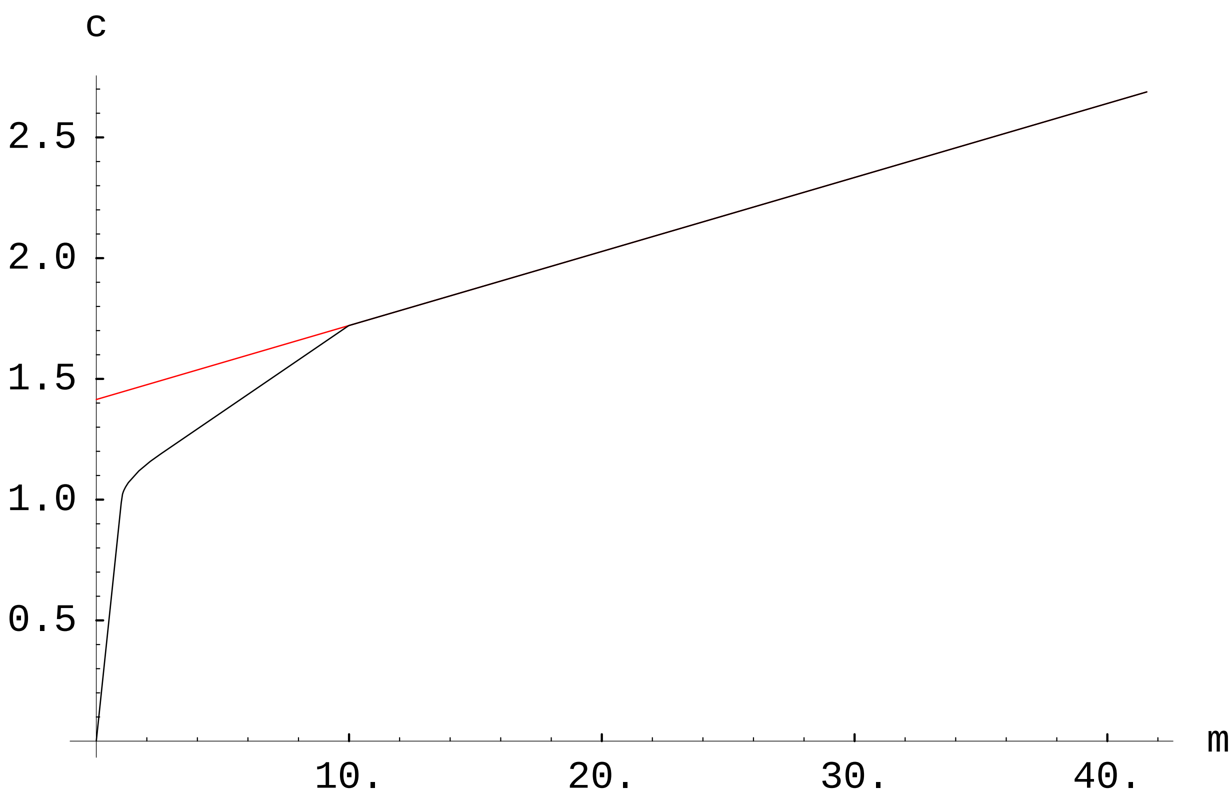
Imposing Constraints Forward



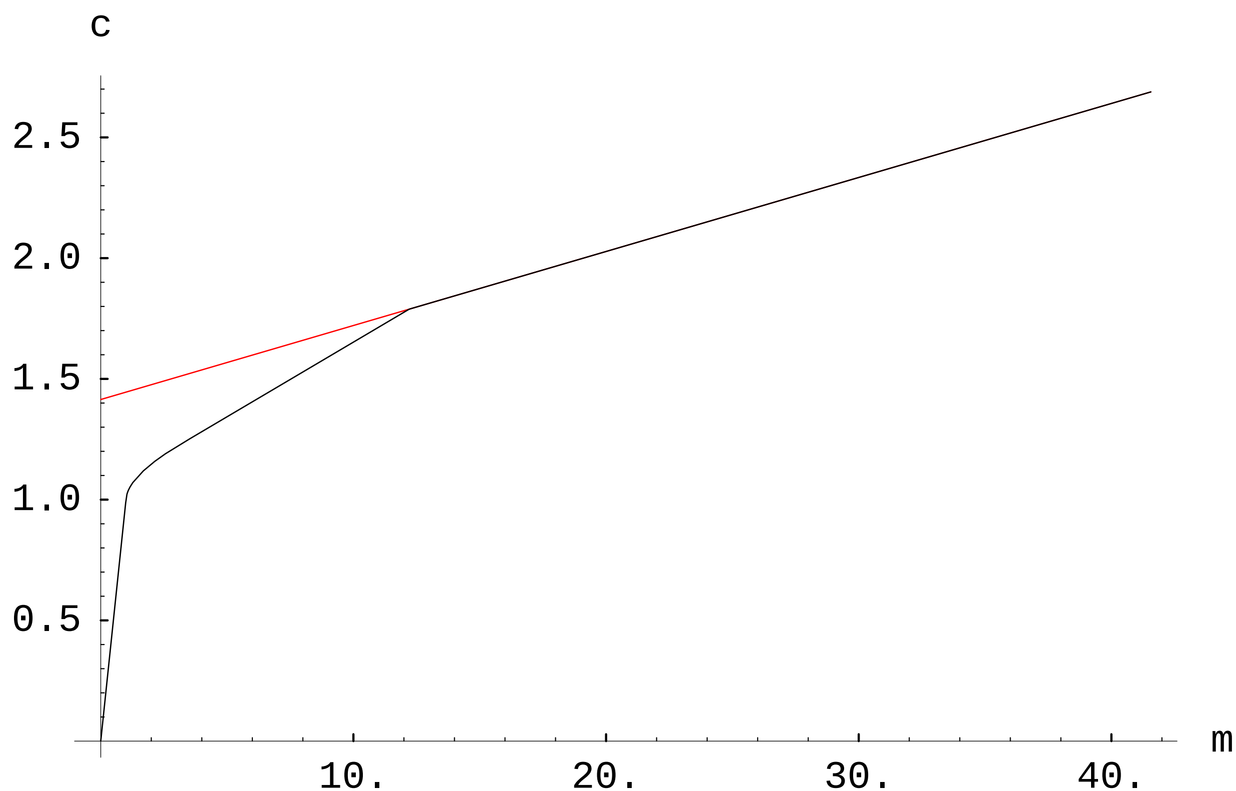
Imposing Constraints Forward



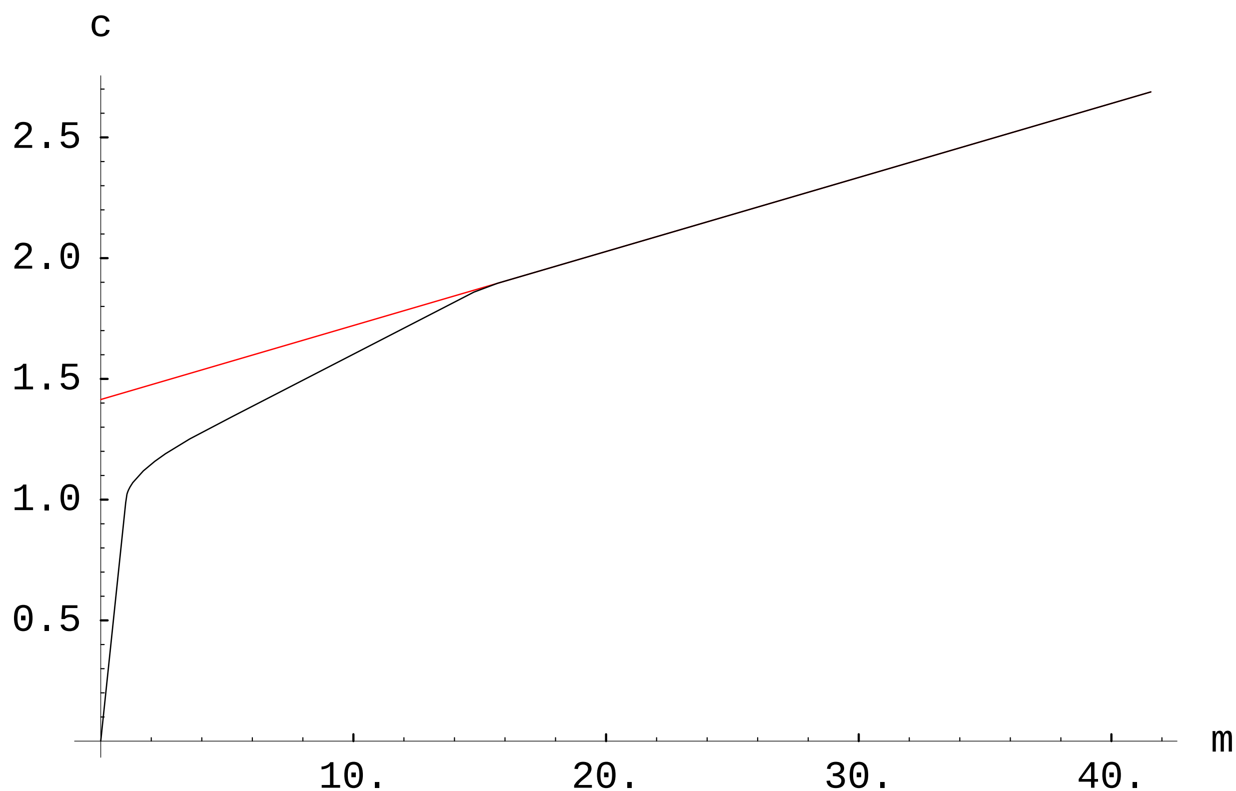
Imposing Constraints Forward



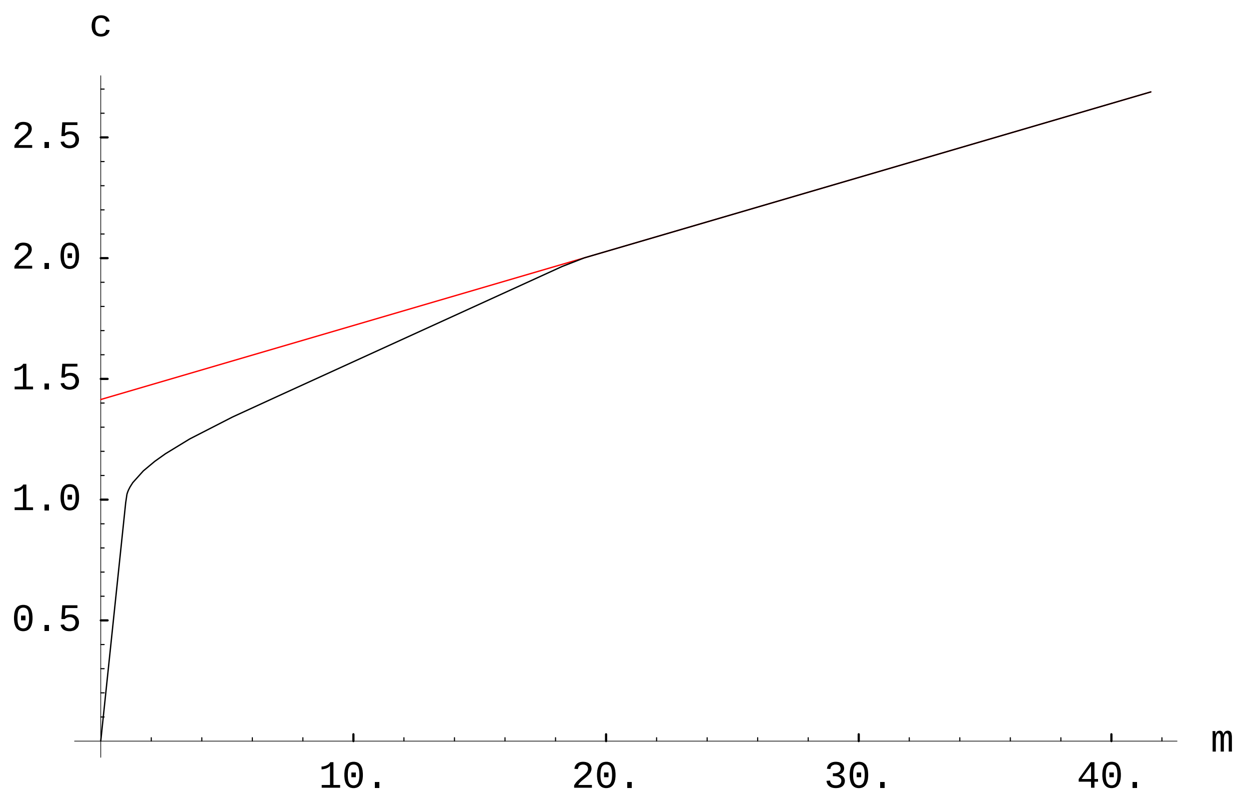
Imposing Constraints Forward



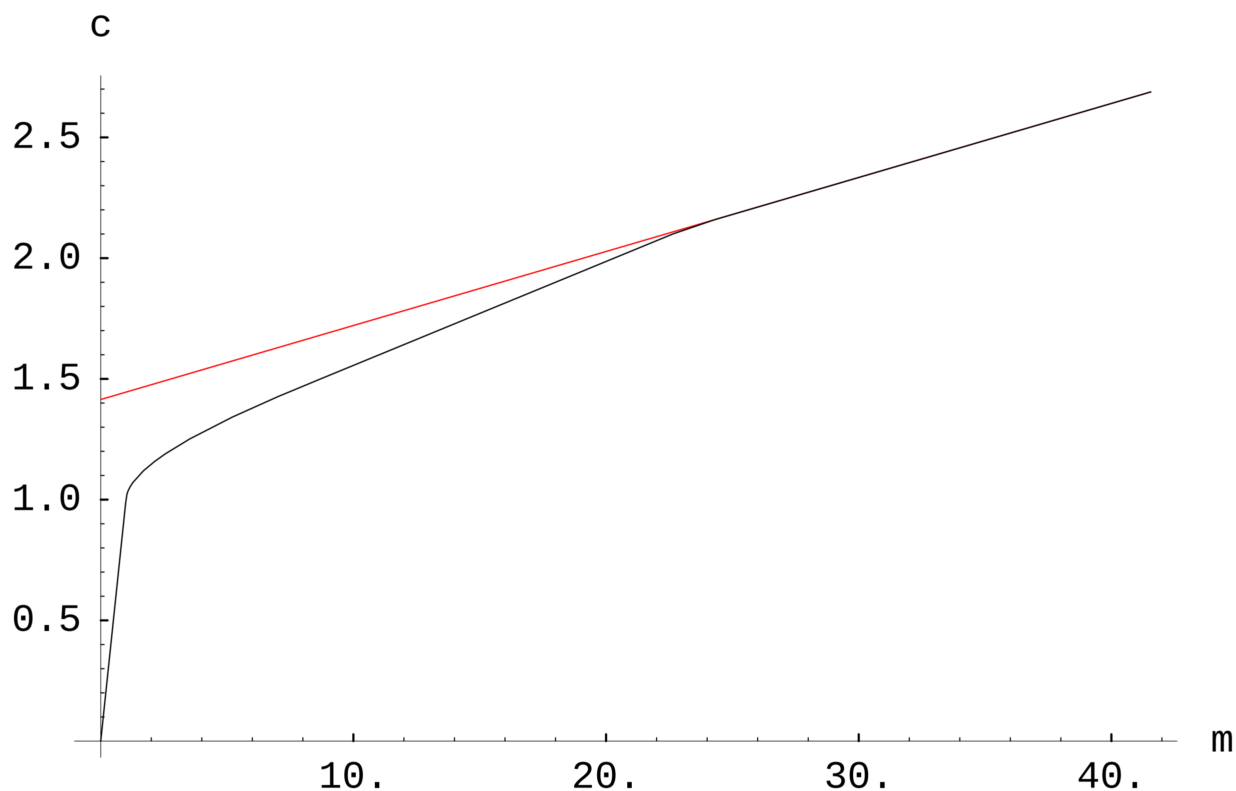
Imposing Constraints Forward



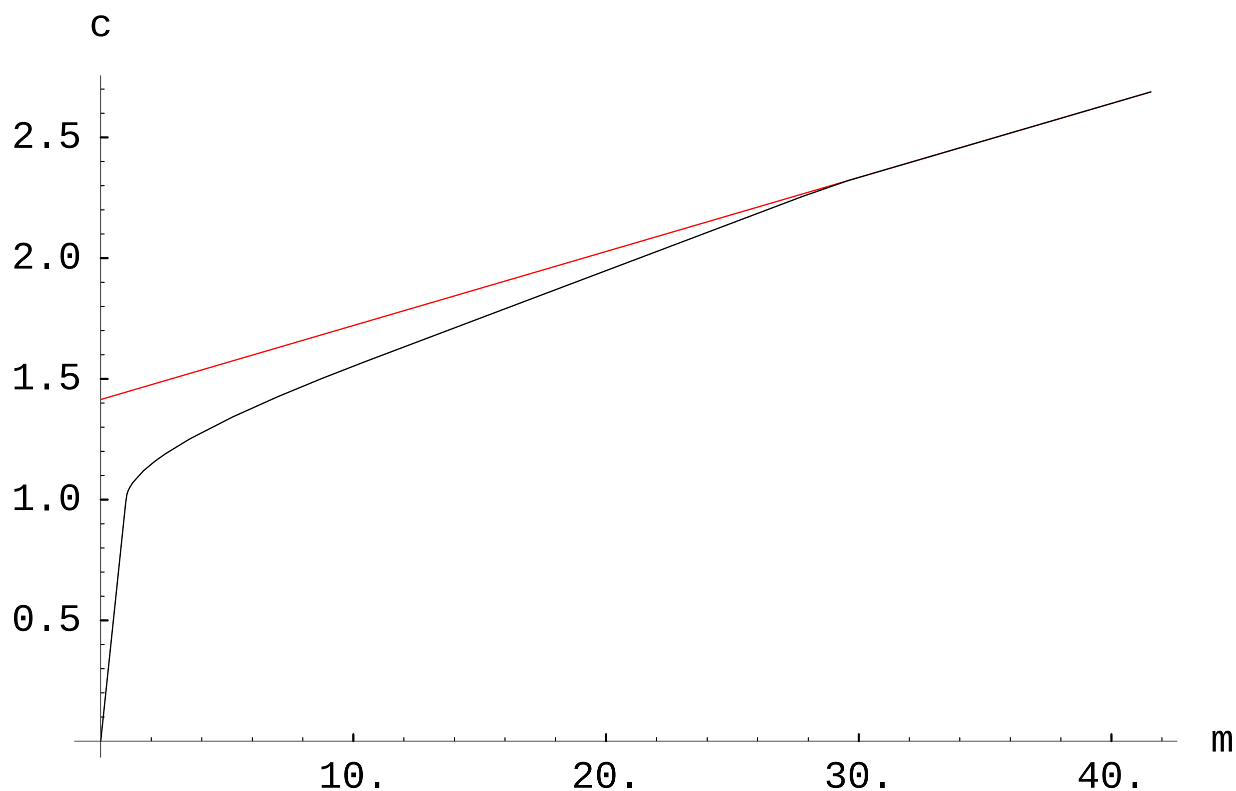
Imposing Constraints Forward



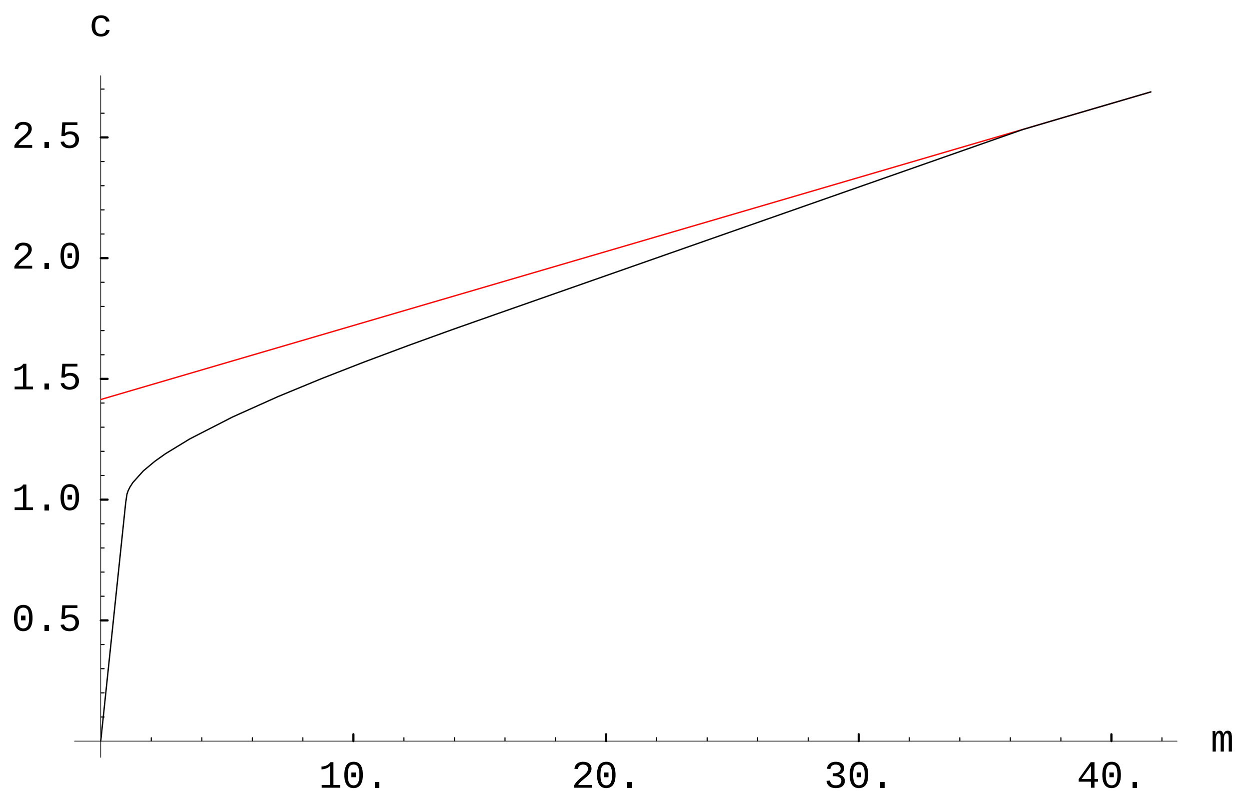
Imposing Constraints Forward



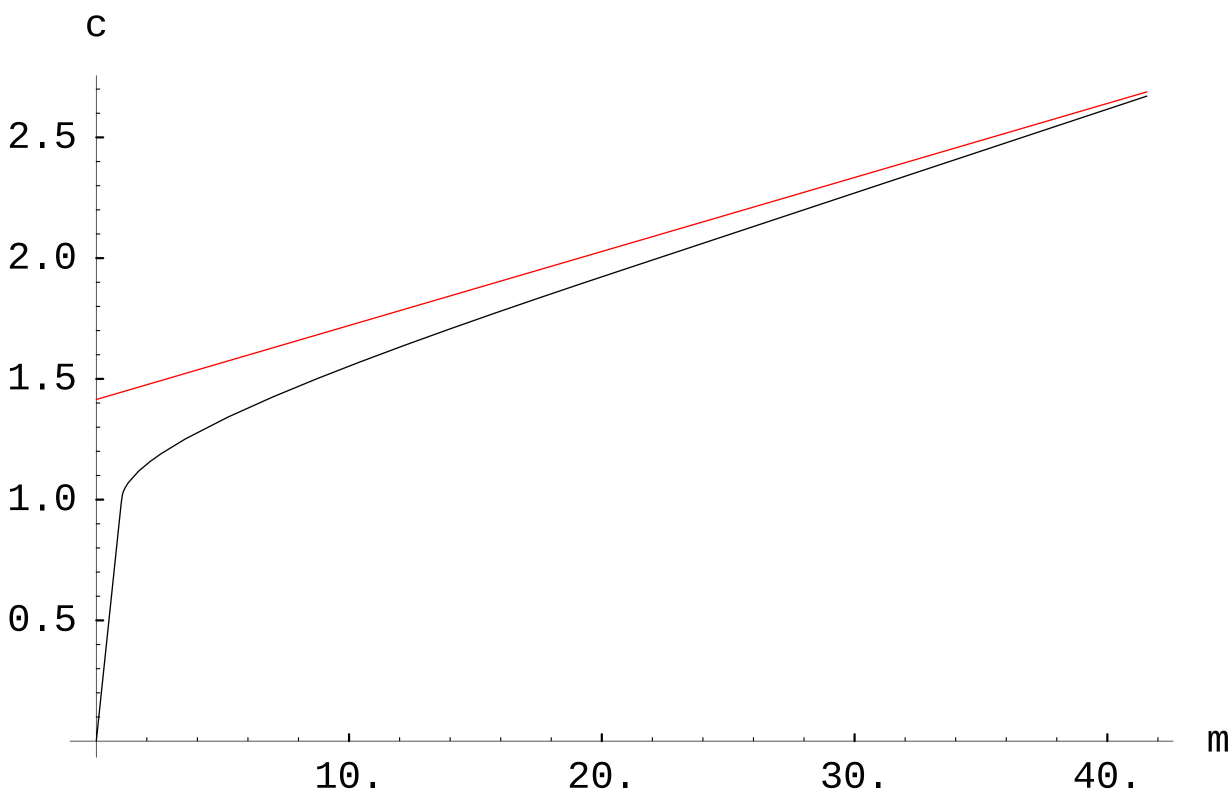
Imposing Constraints Forward



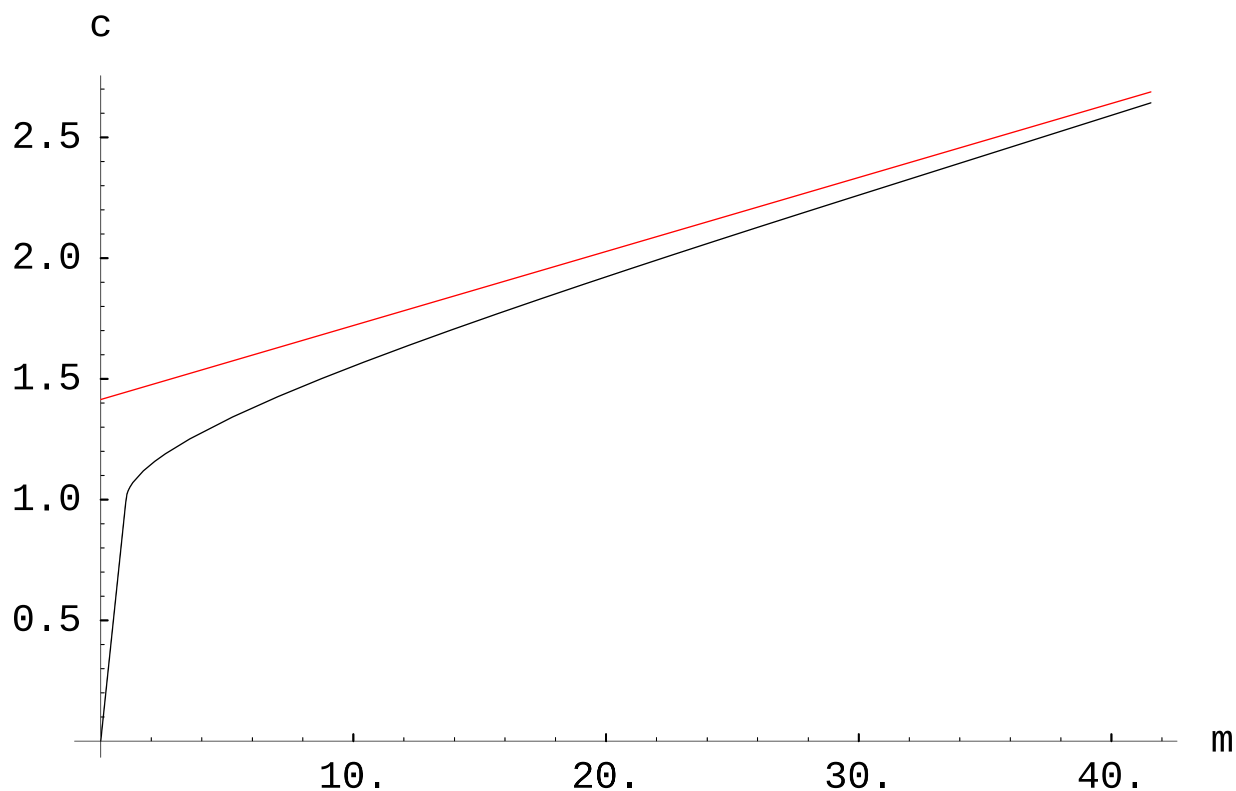
Imposing Constraints Forward



Imposing Constraints Forward



Imposing Constraints Forward



Risks ≈ Constraints

Period $T+1$: $c_{T+1} = m_{T+1}$

Risks ≈ Constraints

Period $T+1$: $c_{T+1} = m_{T+1}$

1. PF constrained consumer: $m_{T+1} = 1 + a_T$

Risks ≈ Constraints

Period $T+1$: $c_{T+1} = m_{T+1}$

1. PF constrained consumer: $m_{T+1} = 1 + a_T$
2. Risky unconstrained consumer:

$$m_{T+1} = \begin{cases} a_T + 1/(1-p) & \text{with prob } (1-p) \\ a_T & \text{with prob } p \end{cases}$$

Risks ≈ Constraints

Period $T+1$: $c_{T+1} = m_{T+1}$

1. PF constrained consumer: $m_{T+1} = 1 + a_T$
2. Risky unconstrained consumer:

$$m_{T+1} = \begin{cases} a_T + 1/(1-p) & \text{with prob } (1-p) \\ a_T & \text{with prob } p \end{cases}$$

Result:

$$\lim_{p \downarrow 0} \tilde{\mathbf{a}}_{T,T}(m_T) = \mathbf{a}_{T,T}(m_T)$$

.

Positive Result 1

Theorem 3 *Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ increases precautionary saving more for a perfect foresight consumer who faces $n+1$ relevant liquidity constraints in* $\mathbf{T}_t$ *(counting backwards) than for a perfect foresight consumer who faces only n relevant constraints in* $\mathbf{T}_t$*. That is,*

$$\mathbf{c}_{t,T-(q+1)}(m) - \tilde{\mathbf{c}}_{t,T-(q+1)}(m) \geq \mathbf{c}_{t,T-q}(m) - \tilde{\mathbf{c}}_{t,T-q}(m)$$

.

Negative Result 1

Consider two different sets of dates at which constraints apply, $\mathbf{T}_t$ and $\hat{\mathbf{T}}_t$, where $\hat{\mathbf{T}}_t$ is a strict superset of $\mathbf{T}_t$. Indicate the consumption function for the consumer who faces the extra constraints by $\hat{\mathbf{c}}_{t,\bullet}$.

Negative Result 1

Consider two different sets of dates at which constraints apply, $\mathbf{T}_t$ and $\hat{\mathbf{T}}_t$, where $\hat{\mathbf{T}}_t$ is a strict superset of $\mathbf{T}_t$. Indicate the consumption function for the consumer who faces the extra constraints by $\hat{\mathbf{c}}_{t,\bullet}$.

Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ does not *necessarily increase precautionary saving more for the consumer facing a larger number of future constraints. That is,*

$$\mathbf{c}_{t,T-n}(m) - \tilde{\mathbf{c}}_{t,T-n}(m) \lesseqgtr \hat{\mathbf{c}}_{t,T-n}(m) - \tilde{\hat{\mathbf{c}}}_{t,T-n}(m)$$

.

Negative Result 2

Consider two different sets of dates at which risks apply, $\mathbf{Q}_t$ and $\hat{\mathbf{Q}}_t$, where $\hat{\mathbf{Q}}_t$ is a strict superset of $\mathbf{Q}_t$. Indicate the consumption function for the consumer who faces the extra risk(s) by $\hat{\mathbf{c}}_{t,\bullet}$.

Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ does not *necessarily increase precautionary saving more, at a given m, than for the consumer facing a larger number of future risks. That is,*

$$\mathbf{c}_{t,T-n}(m) - \tilde{\mathbf{c}}_{t,T-n}(m) \lesseqgtr \hat{\mathbf{c}}_{t,T-n}(m) - \tilde{\hat{\mathbf{c}}}_{t,T-n}(m)$$

Negative Result 2

Consider two different sets of dates at which risks apply, $\mathbf{Q}_t$ and $\hat{\mathbf{Q}}_t$, where $\hat{\mathbf{Q}}_t$ is a strict superset of $\mathbf{Q}_t$. Indicate the consumption function for the consumer who faces the extra risk(s) by $\hat{\mathbf{c}}_{t,\bullet}$.

Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ does not *necessarily increase precautionary saving more, at a given m, than for the consumer facing a larger number of future risks. That is,*

$$\mathbf{c}_{t,T-n}(m) - \tilde{\mathbf{c}}_{t,T-n}(m) \lesseqgtr \hat{\mathbf{c}}_{t,T-n}(m) - \tilde{\hat{\mathbf{c}}}_{t,T-n}(m)$$

This can be seen from the previous fact and from the essential equivalence of constraints and risks. .

Positive Result 2

Define as ‘blighted’ a consumer who faces some combination of future risks and future constraints; the unconstrained perfect foresight consumer with the same horizon is unblighted. Indicate the consumption function for the blighted consumer as $\hat{\mathbf{c}}_{t,\bullet}$. Our final result can be stated as

Positive Result 2

Define as 'blighted' a consumer who faces some combination of future risks and future constraints; the unconstrained perfect foresight consumer with the same horizon is unblighted. Indicate the consumption function for the blighted consumer as $\hat{\mathbf{c}}_{t,\bullet}$. Our final result can be stated as

Introduction of a risk ξ_{t+1} that is realized between t and $t+1$ increases precautionary saving more, at a given m, for the blighted than for the unblighted consumer. That is,

$$\hat{\mathbf{c}}_{t,T-n}(m) - \tilde{\hat{\mathbf{c}}}_{t,T-n}(m) \geq \mathbf{c}_{t,T}(m) - \tilde{\mathbf{c}}_{t,T}(m)$$

.

Conclusions

Effects of future risks and future constraints are very similar

Conclusions

Effects of future risks and future constraints are very similar

- Imposition of a new constraint or risk unambiguously reduces $\mathbf{c}_t$

Conclusions

Effects of future risks and future constraints are very similar

- Imposition of a new constraint or risk unambiguously reduces $\mathbf{c}_t$
- Precautionary effect of adding a new constraint or a new risk depends on CCC:

Conclusions

Effects of future risks and future constraints are very similar

- Imposition of a new constraint or risk unambiguously reduces $\mathbf{c}_t$
- Precautionary effect of adding a new constraint or a new risk depends on CCC:
  - If modified $\hat{c}(m)$ is a CCC of baseline $c(m)$, prudence rises globally

Conclusions

Effects of future risks and future constraints are very similar

- Imposition of a new constraint or risk unambiguously reduces $\mathbf{c}_t$
- Precautionary effect of adding a new constraint or a new risk depends on CCC:
  - If modified $\hat{c}(m)$ is a CCC of baseline $c(m)$, prudence rises globally
- Otherwise prudence may be higher at some m, lower at others

Conclusions

Effects of future risks and future constraints are very similar

- Imposition of a new constraint or risk unambiguously reduces $\mathbf{c}_t$
- Precautionary effect of adding a new constraint or a new risk depends on CCC:
  - If modified $\hat{c}(m)$ is a CCC of baseline $c(m)$, prudence rises globally
- Otherwise prudence may be higher at some m, lower at others
  - Future risks/constraints can 'hide' effect of current risks/constraints